A NOTE ON THE STANDARD MODEL IN A GRAVITATION FIELD.

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Abstract. The Standard Model of elementary particles is a theory unifying three of the four basic forces of the Nature: electromagnetic, weak, and strong interactions. In this paper we consider the Standard Model in the presence of a classical (non-quantized) gravitation field and apply a bundle approach for describing it.

1. Fermion fields of the Standard Model.

Fermion fields of the Standard Model are subdivided into two parts: lepton fields and quark fields. Lepton fields are subdivided into three generations. The first generation is represented by an electron e and an electronic neutrino ν_e, the second generation is represented by a muon μ and its neutrino ν_μ, and the third generation is represented by a tauon τ and its neutrino ν_τ.

1-st generation	2-nd generation	3-rd generation
e-neutrino ν_e	μ-neutrino ν_μ	τ-neutrino ν_τ
electron e	muon μ	tauon τ

(1.1)

In a similar way, quarks are subdivided into three generations. They are represented in the following table similar to the above table (1.1):

1-st generation	2-nd generation	3-rd generation
up-quark u	charm-quark c	top-quark t
down-quark d	strange-quark s	bottom-quark b

(1.2)

Leptons participate in electromagnetic and weak interactions. These interactions are described by the $\mathrm{U}(1)\times\mathrm{SU}(2)$ symmetry which is spontaneously broken according to the Higgs mechanism. Moreover, they break the chiral symmetry on the level of Dirac spinors. This symmetry is often called the left-to-right symmetry, but we prefer to say the chiral-to-antichiral symmetry or simply the chiral symmetry (see some details in [1]). We distinguish between lepton wave functions by mens of the generation index enclosed into square brackets:

$$\boldsymbol{\psi}[e], \qquad\qquad \boldsymbol{\psi}[\mu], \qquad\qquad \boldsymbol{\psi}[\tau]. \tag{1.3}$$

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The wave functions (1.3) have chiral and antichiral constituent parts. Chiral parts are doublets with respect to SU(2) symmetry:

$$\overset{\bullet}{\psi}{}^{a\alpha}_{111}[e], \qquad\qquad \overset{\bullet}{\psi}{}^{a\alpha}_{111}[\mu], \qquad\qquad \overset{\bullet}{\psi}{}^{a\alpha}_{111}[\tau]. \tag{1.4}$$

The indices in (1.4) means that we take the space-time manifold M equipped the appropriate complex vector bundles DM, $\not{U}M$, and $S\!\not{U}M$ (see [2]). The index $a = 1, 2, 3, 4$ in (1.4) is a spinor index associated with the Dirac bundle DM or, more precisely, with some frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2,\,\mathbf{\Psi}_3,\,\mathbf{\Psi}_4)$ of DM. The index $\alpha = 1, 2$ is associated with some frame $(U,\,\mathbf{\Psi}_1,\,\mathbf{\Psi}_2)$ of the two-dimensional complex bundle $S\!\not{U}M$. Due to the presence of this index the wave functions (1.4) are said to be SU(2)-doublets. Three lower indices in (1.4) are always equal to unity because they are associated with some frame $(U,\,\mathbf{\Psi}_1)$ of the one-dimensional bundle $\not{U}M$.

The antichiral parts of the wave functions (1.3) are SU(2)-singlets. Their components have one spinor index a and six U(1) indices equal to 1:

$$\overset{\circ}{\psi}{}^{a}_{111111}[e], \qquad\qquad \overset{\circ}{\psi}{}^{a}_{111111}[\mu], \qquad\qquad \overset{\circ}{\psi}{}^{a}_{111111}[\tau]. \tag{1.5}$$

By usual convention (see [3]) antichiral (right) neutrinos are not considered. In this paper we follow this convention, though in some papers right neutrinos are introduced, e. g. in [4].

The wave functions (1.4) and (1.5) are chiral and antichiral in the sense of the following equalities relating them with the components of the chirality operator $\mathbf{H}$:

$$\sum_{a=1}^{4} H^b_a\,\overset{\bullet}{\psi}{}^{a\alpha}_{111}[q] = \overset{\bullet}{\psi}{}^{b\alpha}_{111}[e], \qquad\qquad \sum_{a=1}^{4} H^b_a\,\overset{\circ}{\psi}{}^{a}_{111111}[q] = -\overset{\circ}{\psi}{}^{b}_{111111}[q]. \tag{1.6}$$

Here $q = e, \mu, \tau$ is the generation index. The chirality operator $\mathbf{H}$ is an attribute of the Dirac bundle (see details in [1]). In physical literature (see [3] as an example), when the flat Minkowski space is taken for the space-time manifold M, the chirality operator $\mathbf{H}$ is represented by the Dirac matrix γ^5

$$H^b_a = \begin{Vmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{Vmatrix} = \gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3. \tag{1.7}$$

Other Dirac γ-matrices are given by the following formulas (see (1.13) in [5]):

$$\begin{aligned} &\gamma^{b\,0}_a = \begin{Vmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{Vmatrix}, &&\gamma^{b1}_a = \begin{Vmatrix} 0 & 0 & 0 & -1\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 \end{Vmatrix},\\ &\gamma^{b\,2}_a = \begin{Vmatrix} 0 & 0 & 0 & i\\ 0 & 0 & -i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0 \end{Vmatrix}, &&\gamma^{b\,3}_a = \begin{Vmatrix} 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 \end{Vmatrix}. \end{aligned} \tag{1.8}$$

By means of the chirality operator $\mathbf{H}$ we define two projection operators:

$$\overset{\bullet}{\mathbf{H}} = \frac{\mathbf{id} + \mathbf{H}}{2}, \qquad \overset{\circ}{\mathbf{H}} = \frac{\mathbf{id} - \mathbf{H}}{2}. \tag{1.9}$$

Here $\mathbf{id}$ is the identity operator. Therefore, the components of the projection operators (1.9) are given by the formulas

$$\overset{\bullet}{H}{}^{b}_{a} = \frac{\delta^{b}_{a} + H^{b}_{a}}{2}, \qquad \overset{\circ}{H}{}^{b}_{a} = \frac{\delta^{b}_{a} - H^{b}_{a}}{2}. \tag{1.10}$$

By means of (1.10) the equalities (1.6) are written as follows:

$$\sum_{a=1}^{4} \overset{\circ}{H}{}^{b}_{a} \, \overset{\bullet}{\psi}{}^{a\alpha}_{111}[q] = 0, \qquad \sum_{a=1}^{4} \overset{\bullet}{H}{}^{b}_{a} \, \overset{\circ}{\psi}{}^{a}_{111111}[q] = 0. \tag{1.11}$$

The indices a and b in (1.6), (1.7), (1.8), (1.9), (1.10), and (1.11) are spinor indices. They are associated with the Dirac bundle DM. The third index of the Dirac matrices represented by the numbers $0, 1, 2, 3$ in (1.8) is a spacial index, it is associated with the tangent bundle TM.

Like in the case of leptons, quark wave functions are subdivided into three generations according to the generation table (1.2):

$$\boldsymbol{\psi}[1], \qquad \boldsymbol{\psi}[2], \qquad \boldsymbol{\psi}[3]. \tag{1.12}$$

However, now we use a numeric index for generations, since $\boldsymbol{\psi}[1]$ describes both an up-quark and a down-quark. Similarly, $\boldsymbol{\psi}[2]$ describes a charm-quark together with a strange-quark and $\boldsymbol{\psi}[3]$ describes a top-quark together with a bottom-quark. Chiral and antichiral parts of the wave functions (1.12) behave differently with respect to the SU(2) symmetry. Chiral parts are SU(2)-doublets:

$$\overset{\bullet}{\psi}{}^{a1\alpha\beta}[1], \qquad \overset{\bullet}{\psi}{}^{a1\alpha\beta}[2], \qquad \overset{\bullet}{\psi}{}^{a1\alpha\beta}[3]. \tag{1.13}$$

Note that in (1.13), in contrast to (1.4), we have one more index. The additional index $\beta = 1, 2, 3$ is responsible for color, it describes strong interactions of quarks. For antichiral parts of the wave functions (1.12) the index α is omitted:

$$\begin{gathered} \overset{\circ}{\psi}{}^{a1111\beta}[u], \qquad \overset{\circ}{\psi}{}^{a1111\beta}[c], \qquad \overset{\circ}{\psi}{}^{a1111\beta}[t], \\ \overset{\circ}{\psi}{}^{a\beta}_{11}[d], \qquad \overset{\circ}{\psi}{}^{a\beta}_{11}[s], \qquad \overset{\circ}{\psi}{}^{a\beta}_{11}[b]. \end{gathered} \tag{1.14}$$

They are SU(2)-singlets. The wave functions (1.13) and (1.14) are chiral and antichiral in the sense of the following equalities:

$$\begin{aligned} &\sum_{a=1}^{4} H^{b}_{a} \, \overset{\bullet}{\psi}{}^{a1\alpha\beta}[q] = \overset{\bullet}{\psi}{}^{b1\alpha\beta}[q], && q = 1, 2, 3; \\ &\sum_{a=1}^{4} H^{b}_{a} \, \overset{\circ}{\psi}{}^{a1111\beta}[q] = -\overset{\circ}{\psi}{}^{b1111\beta}[q], && q = u, c, t; \\ &\sum_{a=1}^{4} H^{b}_{a} \, \overset{\circ}{\psi}{}^{a\beta}_{11}[q] = -\overset{\circ}{\psi}{}^{b\beta}_{11}[q], && q = d, s, b. \end{aligned} \tag{1.15}$$

The equalities (1.15) are analogous to (1.6). In terms of the projection operators introduced by the formulas (1.9) they are rewritten as

$$\sum_{a=1}^{4}\overset{\circ}{H}{}^{b}_{a}\,\overset{\bullet}{\psi}{}^{a1\alpha\beta}[q]=0,\qquad \sum_{a=1}^{4}\overset{\bullet}{H}{}^{b}_{a}\,\overset{\circ}{\psi}{}^{a1111\beta}[q]=0,\qquad \sum_{a=1}^{4}\overset{\bullet}{H}{}^{b}_{a}\,\overset{\circ}{\psi}{}^{\alpha\beta}_{11}[q]=0. \tag{1.16}$$

In this form (1.16) the identities (1.15) are analogous to the identities (1.11).

2. The Higgs field and the classical vacuum.

The classical Higgs field $\boldsymbol{\varphi}$ is a scalar field being SU(2)-doublet with respect to electro-weak bundle SUM. In a coordinate form, i. e. upon choosing some frame $(U,\,\boldsymbol{\Psi}_1,\,\boldsymbol{\Psi}_2)$ of SUM and some frame $(U,\,\boldsymbol{\Psi}_1)$ of UM, it is represented as

$$\varphi^{\alpha 111},\ \text{ where }\ \alpha=1,2. \tag{2.1}$$

Let $(U,\,\boldsymbol{\Upsilon}_0,\,\boldsymbol{\Upsilon}_1,\,\boldsymbol{\Upsilon}_2,\,\boldsymbol{\Upsilon}_3)$ be some frame of the tangent bundle TM and let $L_{\boldsymbol{\Upsilon}_k}$ be the Lie derivative along the vector field $\boldsymbol{\Upsilon}_k$. Then we can define a covariant differentiation of the Higgs field $\boldsymbol{\varphi}$ through the following formula:

$$\nabla_k\,\varphi^{\alpha 111}=L_{\boldsymbol{\Upsilon}_k}(\varphi^{\alpha 111})+\sum_{\theta=1}^{2}\mathbb{A}^{\alpha}_{k\theta}\,\varphi^{\theta 111}+3\,\mathrm{A}^{1}_{k1}\,\varphi^{\alpha 111}. \tag{2.2}$$

Covariant differentiations for the wave functions of leptons and quarks (1.4), (1.5), (1.13), and (1.14) are defined in a similar way:

$$\begin{gathered}\nabla_k\,\overset{\bullet}{\psi}{}^{a\alpha}_{111}[q]=L_{\boldsymbol{\Upsilon}_k}(\overset{\bullet}{\psi}{}^{a\alpha}_{111}[q])+\sum_{b=1}^{4}\mathrm{A}^{a}_{kb}\ \overset{\bullet}{\psi}{}^{b\alpha}_{111}[q]\,+\\ +\sum_{\theta=1}^{2}\mathbb{A}^{\alpha}_{k\theta}\,\overset{\bullet}{\psi}{}^{a\theta}_{111}[q]-3\,\mathrm{A}^{1}_{k1}\,\overset{\bullet}{\psi}{}^{a\alpha}_{111}[q],\ \text{ where }\ q=e,\mu,\tau;\end{gathered} \tag{2.3}$$

$$\begin{gathered}\nabla_k\,\overset{\circ}{\psi}{}^{a}_{111111}[q]=L_{\boldsymbol{\Upsilon}_k}(\overset{\circ}{\psi}{}^{a}_{111111}[q])+\sum_{b=1}^{4}\mathrm{A}^{a}_{kb}\ \overset{\circ}{\psi}{}^{b}_{111111}[q]\,-\\ -\,6\,\mathrm{A}^{1}_{k1}\,\overset{\circ}{\psi}{}^{a}_{111111}[q],\ \text{ where }\ q=e,\mu,\tau;\end{gathered} \tag{2.4}$$

$$\begin{gathered}\nabla_k\,\overset{\bullet}{\psi}{}^{a1\alpha\beta}[q]=L_{\boldsymbol{\Upsilon}_k}(\overset{\bullet}{\psi}{}^{a1\alpha\beta}[q])+\sum_{b=1}^{4}\mathrm{A}^{a}_{kb}\ \overset{\bullet}{\psi}{}^{b1\alpha\beta}[q]+\mathrm{A}^{1}_{k1}\,\overset{\bullet}{\psi}{}^{a1\alpha\beta}[q]\,+\\ +\sum_{\theta=1}^{2}\mathbb{A}^{\alpha}_{k\theta}\,\overset{\bullet}{\psi}{}^{a1\theta\beta}[q]+\sum_{\theta=1}^{3}\mathbf{A}^{\alpha}_{k\theta}\,\overset{\bullet}{\psi}{}^{a1\alpha\theta}[q],\ \text{ where }\ q=1,2,3;\end{gathered} \tag{2.5}$$

$$\begin{gathered}\nabla_k\,\overset{\circ}{\psi}{}^{a1111\beta}[q]=L_{\boldsymbol{\Upsilon}_k}(\overset{\circ}{\psi}{}^{a1111\beta}[q])+\sum_{b=1}^{4}\mathrm{A}^{a}_{kb}\ \overset{\circ}{\psi}{}^{b1111\beta}[q]\,+\\ +\,4\,\mathrm{A}^{1}_{k1}\,\overset{\circ}{\psi}{}^{a1111\beta}[q]+\sum_{\theta=1}^{3}\mathbf{A}^{\alpha}_{k\theta}\,\overset{\circ}{\psi}{}^{a1111\theta}[q],\ \text{ where }\ q=u,c,t;\end{gathered} \tag{2.6}$$

$$\begin{aligned}\nabla_k\,\overset{\circ}{\psi}{}^{a\beta}_{11}[q]&=L_{\Upsilon_k}(\overset{\circ}{\psi}{}^{a\beta}_{11}[q])+\sum_{b=1}^{4}\mathrm{A}^a_{kb}\,\overset{\circ}{\psi}{}^{b\beta}_{11}[q]-\\&-2\,\mathrm{A}^1_{k1}\,\overset{\circ}{\psi}{}^{a\beta}_{11}[q]+\sum_{\theta=1}^{3}\mathbb{A}^{\alpha}_{k\theta}\,\overset{\circ}{\psi}{}^{a\theta}_{11}[q],\ \text{ where }\ q=d,s,b;\end{aligned}\tag{2.7}$$

Here $\mathbb{A}^{\alpha}_{k\theta}$ and A^1_{k1} are the components of some connections associated with the electro-weak bundles SUM and UM, while $\mathbb{A}^{\alpha}_{k\theta}$ are the components of some connection associated with the color bundle SUM. In Standard Model they are interpreted as the components of gauge fields.

Apart from the gauge field connections, in (2.3), (2.4), (2.5), (2.6), and (2.7) we see the components A^a_{kb} of the spinor connection. These quantities are due to the presence of a gravitation field. Unlike A^1_{k1}, $\mathbb{A}^{\alpha}_{k\theta}$, and $\mathbb{A}^{\alpha}_{k\theta}$, the spinor connection components A^a_{kb} are not interpreted as gauge fields. They are uniquely determined by the metric tensor $\mathbf{g}$ in the base space-time manifold M (see [7]).

The gauge field connections A, $\mathbb{A}$, and $\mathbb{A}$ are related to the basic fields in UM, SUM, and SUM through the series of concordance conditions (see [6]):

$$\nabla\mathbf{D}=0,\tag{2.8}$$

$$\nabla\mathbf{D}=0,\qquad\qquad \nabla\mathbf{d}=0,\tag{2.9}$$

$$\nabla\mathbf{D}=0,\qquad\qquad \nabla\mathbf{d}=0.\tag{2.10}$$

In a coordinate form the concordance conditions (2.8), (2.9), (2.10) are written as

$$\nabla_k\,D_{11}=L_{\Upsilon_k}(D_{11})-D_{11}\,\mathrm{A}^1_{k1}-D_{11}\,\overline{\mathrm{A}^1_{k1}}=0,\tag{2.11}$$

$$\nabla_k\,D_{i\bar{j}}=L_{\Upsilon_k}(D_{i\bar{j}})-\sum_{a=1}^{2}D_{a\bar{j}}\,\mathbb{A}^a_{ki}-\sum_{\bar{a}=1}^{2}D_{i\bar{a}}\,\overline{\mathbb{A}^{\bar{a}}_{k\bar{j}}}=0,\tag{2.12}$$

$$\nabla_k\,d_{ij}=L_{\Upsilon_k}(d_{ij})-\sum_{a=1}^{2}d_{aj}\,\mathbb{A}^a_{ki}-\sum_{a=1}^{2}d_{ia}\,\mathbb{A}^a_{kj}=0,\tag{2.13}$$

$$\nabla_k\,D_{i\bar{j}}=L_{\Upsilon_k}(D_{i\bar{j}})-\sum_{a=1}^{3}D_{a\bar{j}}\,\mathbb{A}^a_{ki}-\sum_{\bar{a}=1}^{3}D_{i\bar{a}}\,\overline{\mathbb{A}^{\bar{a}}_{k\bar{j}}}=0,\tag{2.14}$$

$$\begin{aligned}\nabla_k\,d_{ijm}&=L_{\Upsilon_k}(d_{ijm})-\sum_{a=1}^{3}d_{ajm}\,\mathbb{A}^a_{ki}-\sum_{a=1}^{3}d_{iam}\,\mathbb{A}^a_{kj}-\\&-\sum_{a=1}^{3}d_{ija}\,\mathbb{A}^a_{km}=0.\end{aligned}\tag{2.15}$$

Apart from (2.11), (2.12), (2.13), (2.14), (2.15), the tensor fields $\mathbf{D}$ and $\mathbf{D}$ are related to $\mathbf{d}$ and $\mathbf{d}$ by means of the following concordance conditions (see [2]):

$$\sum_{i=1}^{2}\sum_{j=1}^{2}d^{ij}\,D_{i\bar{i}}\,D_{j\bar{j}}=-\overline{d_{\bar{i}\bar{j}}},\tag{2.16}$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}d^{ijk}\,D_{i\bar{i}}\,D_{j\bar{j}}\,D_{k\bar{k}}=\overline{d_{\bar{i}\bar{j}\bar{k}}}.$$

Each connection produces its curvature tensor. In the case of the gauge connections $\mathbb{A}$, $\mathbb{A}$, and $\mathbb{A}$ for the components of the curvature tensors we have

$$\mathfrak{R}^1_{1ij}=\mathfrak{R}_{ij}=L_{\boldsymbol{\Upsilon}_i}(\mathbb{A}^1_{j1})-L_{\boldsymbol{\Upsilon}_j}(\mathbb{A}^1_{i1})-\sum_{h=0}^{3}c^{\,h}_{ij}\,\mathbb{A}^1_{h1},\tag{2.17}$$

$$\begin{aligned}\mathfrak{R}^p_{kij}&=L_{\boldsymbol{\Upsilon}_i}(\mathbb{A}^p_{j\,k})-L_{\boldsymbol{\Upsilon}_j}(\mathbb{A}^p_{i\,k})+\\&+\sum_{h=1}^{2}\Bigl(\mathbb{A}^p_{i\,h}\,\mathbb{A}^h_{j\,k}-\mathbb{A}^p_{j\,h}\,\mathbb{A}^h_{i\,k}\Bigr)-\sum_{h=0}^{3}c^{\,h}_{ij}\,\mathbb{A}^p_{hk},\end{aligned}\tag{2.18}$$

$$\begin{aligned}\mathfrak{R}^p_{kij}&=L_{\boldsymbol{\Upsilon}_i}(\mathbb{A}^p_{j\,k})-L_{\boldsymbol{\Upsilon}_j}(\mathbb{A}^p_{i\,k})+\\&+\sum_{h=1}^{3}\Bigl(\mathbb{A}^p_{i\,h}\,\mathbb{A}^h_{j\,k}-\mathbb{A}^p_{j\,h}\,\mathbb{A}^h_{i\,k}\Bigr)-\sum_{h=0}^{3}c^{\,h}_{ij}\,\mathbb{A}^p_{hk}\end{aligned}\tag{2.19}$$

(see [6]). Here $c^{\,h}_{ij}$ are the parameters introduced through the commutation relationships for the frame vector fields $\boldsymbol{\Upsilon}_0$, $\boldsymbol{\Upsilon}_1$, $\boldsymbol{\Upsilon}_2$, $\boldsymbol{\Upsilon}_3$:

$$[\boldsymbol{\Upsilon}_i,\,\boldsymbol{\Upsilon}_j]=\sum_{k=0}^{3}c^{\,k}_{ij}\,\boldsymbol{\Upsilon}_k.\tag{2.20}$$

From (2.11), (2.12), and (2.14) we derive the following identities for the curvature tensors introduced by the formulas (2.17), (2.18), and (2.19):

$$\mathfrak{R}^1_{1ij}+\overline{\mathfrak{R}^1_{1ij}}=0,\tag{2.21}$$

$$\sum_{\bar k=1}^{2}\mathfrak{R}^k_{pij}\,D_{k\bar q}+\sum_{k=1}^{2}\overline{\mathfrak{R}^{\bar k}_{\bar q ij}}\,D_{p\bar k}=0,\tag{2.22}$$

$$\sum_{\bar k=1}^{3}\mathfrak{R}^k_{pij}\,D_{k\bar q}+\sum_{k=1}^{3}\overline{\mathfrak{R}^{\bar k}_{\bar q ij}}\,D_{p\bar k}=0.\tag{2.23}$$

Similarly, from (2.13) and (2.15) for (2.18) and (2.19) we derive

$$\sum_{k=1}^{2}\mathfrak{R}^p_{kij}\,d^{kq}+\sum_{k=1}^{2}\mathfrak{R}^q_{kij}\,d^{pk}=0,\tag{2.24}$$

$$\sum_{k=1}^{3}\mathfrak{R}^p_{kij}\,d^{kqm}+\sum_{k=1}^{3}\mathfrak{R}^q_{kij}\,d^{pkm}+\sum_{k=1}^{3}\mathfrak{R}^m_{kij}\,d^{pqk}=0.\tag{2.25}$$

The equality (2.21) means that $\mathfrak{R}^1_{1ij}$ given by the formula (2.17) is a purely imaginary number. The equalities (2.22) and (2.23) are more complicated. They mean that for fixed i and j the components of the curvature tensors (2.18) and (2.19) form skew-Hermitian matrices with respect to Hermitian forms $\mathbf{D}$ and $\mathbf{D}$ respectively.

The equality (2.25) looks rather complicated. However, for each fixed i and j it is equivalent to the following zero trace condition for the curvature tensor (2.19):

$$\sum_{k=1}^{3}\mathfrak{R}^k_{kij}=0.\tag{2.26}$$

From (2.24) one can derive the analogous equality for (2.18):

$$\sum_{k=1}^{2}\mathfrak{R}^{k}_{kij}=0. \tag{2.27}$$

Unlike (2.26), in this case (2.24) is not equivalent to (2.27). Remember that in SUM the tensor $\mathbf{d}$ is used for raising and lowering indices. Let's denote

$$\mathfrak{R}^{pq}_{ij}=\sum_{k=1}^{2}\mathfrak{R}^{p}_{kij}\,d^{kq}, \qquad\qquad \mathfrak{R}_{pqij}=\sum_{k=1}^{2}\mathfrak{R}^{k}_{qij}\,d_{kq}. \tag{2.28}$$

In terms of (2.28) the equality (2.24) is equivalent to the symmetry conditions

$$\mathfrak{R}^{pq}_{ij}=\mathfrak{R}^{qp}_{ij}, \qquad\qquad \mathfrak{R}_{pqij}=\mathfrak{R}_{qpij}.$$

The curvature tensors (2.17), (2.18), (2.19) are physical fields associated with the gauge fields A, A and A. There are the following identities for them:

$$D^{11}\,D_{11}\,\mathfrak{R}^{1}_{1ij}\,\overline{\mathfrak{R}^{1}_{1mn}}=-\mathfrak{R}^{1}_{1ij}\,\mathfrak{R}^{1}_{1mn}, \tag{2.29}$$

$$\sum_{p=1}^{2}\sum_{\bar p=1}^{2}\sum_{q=1}^{2}\sum_{\bar q=1}^{2}D^{q\bar q}\,D_{p\bar p}\,\mathfrak{R}^{p}_{qij}\,\overline{\mathfrak{R}^{\bar p}_{\bar q mn}}=-\sum_{p=1}^{2}\sum_{q=1}^{2}\mathfrak{R}^{p}_{qij}\,\mathfrak{R}^{q}_{pij}, \tag{2.30}$$

$$\sum_{p=1}^{3}\sum_{\bar p=1}^{3}\sum_{q=1}^{3}\sum_{\bar q=1}^{3}D^{q\bar q}\,D_{p\bar p}\,\mathfrak{R}^{p}_{qij}\,\overline{\mathfrak{R}^{\bar p}_{\bar q mn}}=-\sum_{p=1}^{3}\sum_{q=1}^{3}\mathfrak{R}^{p}_{qij}\,\mathfrak{R}^{q}_{pij}. \tag{2.31}$$

By D^{11} in (2.29) we denote the component of the inverse Hermitian metric tensor of the bundle UM. It is related to D_{11} by means of the formula

$$D^{11}=\frac{1}{D_{11}}. \tag{2.32}$$

Similarly, by $D^{q\bar q}$ and $D^{q\bar q}$ in (2.30) and (2.31) we denote the components of the inverse Hermitian metric tensors in SUM and SUM. They form two matrices inverse to the matrices $D_{p\bar p}$ and $D_{p\bar p}$ respectively:

$$\sum_{\bar p=1}^{2}D_{p\bar p}\,D^{q\bar p}=\delta^{q}_{p}, \qquad\qquad \sum_{p=1}^{2}D_{p\bar p}\,D^{p\bar q}=\delta^{\bar q}_{\bar p}, \tag{2.33}$$

$$\sum_{\bar p=1}^{3}D_{p\bar p}\,D^{q\bar p}=\delta^{q}_{p}, \qquad\qquad \sum_{p=1}^{3}D_{p\bar p}\,D^{p\bar q}=\delta^{\bar q}_{\bar p}, \tag{2.34}$$

The formula (2.29) follows immediately from (2.21) and (2.32). The formula (2.30) is derived from (2.22) with the use of the formulas (2.33). Similarly, the formula (2.31) is derived from (2.23) with the use of the formulas (2.34).

Definition 2.1. The electro-weak and color bundles UM, SUM, and SUM are called *flat* if there are three flat connections A, A, and A in these bundles, i.e. three connections with zero curvature tensors (2.17), (2.18), and (2.19).

The connections A, $\mathbb{A}$, and $\mathbb{A}$ in the above definition 2.1 are assumed to be concordant with the basic fields $\mathbf{D}$, $\mathbb{D}$, $\mathbb{D}$, $\mathbb{d}$, and $\mathbb{d}$ of the electro-weak and color bundles. Therefore, the flatness condition

$$\mathfrak{R} = 0 \tag{2.35}$$

written for their curvature tensors (2.17), (2.18), and (2.19) is an additional condition complementary to (2.8), (2.9), and (2.10). Below we implicitly assume that the bundles UM, SUM, and SUM are flat in the sense of the definition 2.1.

Definition 2.2. The components of the flat connections A, $\mathbb{A}$, and $\mathbb{A}$ constitute a classical vacuum of bosonic gauge fields of the Standard Model.

The existence of a classical gauge vacuum is provided by the definition 2.1. The problem of uniqueness of such a vacuum in the case of flat bundles as well as the possibility of non-flat vacua should be studied in separate papers.

From now on let's assume that some flat electro-weak and color bundles UM, SUM, and SUM are chosen and some flat gauge vacuum of them is fixed. Then for general non-vacuum gauge fields we write

$$\mathrm{A}^1_{k1} = \mathrm{A}^1_{k1}[vac] - \frac{i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^1_{k1}, \tag{2.36}$$

$$\mathbb{A}^\alpha_{k\beta} = \mathbb{A}^\alpha_{k\beta}[vac] - \frac{i\,e\,g_2}{\hbar\,c}\,\mathcal{A}^\alpha_{k\beta}. \tag{2.37}$$

$$\mathbb{A}^\alpha_{k\beta} = \mathbb{A}^\alpha_{k\beta}[vac] - \frac{i\,e\,g_3}{\hbar\,c}\,\mathcal{A}^\alpha_{k\beta}. \tag{2.38}$$

Here e is the charge of an electron[1], c is the light speed, and $\hbar$ is the Planck constant. Below are the numeric values of these foundamental constants:

$$\begin{aligned} &e \approx 4.80420440 \cdot 10^{-10}\, g^{1/2} \cdot cm^{3/2} \cdot sec^{-1}, \\ &\hbar \approx 1.05457168 \cdot 10^{-27}\, g \cdot cm^2 \cdot sec^{-1}, \\ &c \approx 2.99792458 \cdot 10^{10}\, cm \cdot sec^{-1}. \end{aligned} \tag{2.39}$$

By g_1, g_2, and g_3 in (2.36), (2.37), and (2.38) we denote some numeric constants which are called the *coupling constants*. Their values are determined a posteriori by comparing the Standard Model predictions with experimental data.

Unlike $\mathrm{A}^\alpha_{k\beta}$, $\mathbb{A}^\alpha_{k\beta}$, and $\mathbb{A}^\alpha_{k\beta}$, the quantities $\mathcal{A}^\alpha_{k\beta}$, $\mathcal{A}^\alpha_{k\beta}$, and $\mathcal{A}^\alpha_{k\beta}$ in (2.36), (2.37), and (2.38) are the components of tensor fields. We denote these tensor fields by $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{A}}$, and $\boldsymbol{\mathcal{A}}$ respectively. Since $\mathfrak{R}[vac] = 0$, substituting (2.36), (2.37), and (2.38) into (2.17), (2.18), and (2.19) respectively, we derive

$$\mathfrak{R}^1_{1ij} = -\frac{i\,e\,g_1}{\hbar\,c}\left(\nabla_i\,\mathcal{A}^1_{j\,1} - \nabla_j\,\mathcal{A}^1_{i\,1}\right) \tag{2.40}$$

$$\mathfrak{R}^p_{kij} = -\frac{i\,e\,g_2}{\hbar\,c}\left(\nabla_i\,\mathcal{A}^p_{j\,k} - \nabla_j\,\mathcal{A}^p_{i\,k}\right) - \sum_{h=1}^{2}\left(\frac{e\,g_2}{\hbar\,c}\right)^2\left(\mathcal{A}^p_{i\,h}\,\mathcal{A}^h_{j\,k} - \mathcal{A}^p_{j\,h}\,\mathcal{A}^h_{i\,k}\right), \tag{2.41}$$

[1] Note that e in (2.36), (2.37), and (2.38) is a positive quantity, therefore, it is better to say that e is the charge of a positron. The numeric values (2.39) of the physical constants e, $\hbar$, and c are taken from the NIST site http://physics.nist.gov/cuu/Constants. The value of e there is given in SI units. It is converted to CGS units by means of the NIST value of ε_0.

$$\mathfrak{R}^p_{kij} = -\frac{i\,e\,g_3}{\hbar\,c}\left(\nabla_i\,\mathcal{A}^p_{j\,k} - \nabla_j\,\mathcal{A}^p_{i\,k}\right) - \sum_{h=1}^{3}\left(\frac{e\,g_3}{\hbar\,c}\right)^2\left(\mathcal{A}^p_{i\,h}\,\mathcal{A}^h_{j\,k} - \mathcal{A}^p_{j\,h}\,\mathcal{A}^h_{i\,k}\right). \tag{2.42}$$

Here ∇_i and ∇_j are vacuum covariant derivatives, i. e. they are calculated with respect to the vacuum electro-weak and color connections. In deriving (2.40), (2.41), and (2.42) from (2.17), (2.18), and (2.19) we used the following equality for the components of the Levi-Civita connection:

$$\Gamma^h_{ij} - \Gamma^h_{ji} = c^h_{ij}. \tag{2.43}$$

The equality (2.43) means that Γ^h_{ij} are the components of a torsion-free (symmetric) connection (see (2.8) in [6]). The quantities c^h_{ij} are introduced by the formula (2.20).

Relying upon the formulas (2.40), (2.41), and (2.42), now we introduce three tensor fields $\mathbf{F}$, F, and $\boldsymbol{\mathsf{F}}$ with the following components:

$$\mathcal{F}_{ij} = \nabla_i\,\mathcal{A}^1_{j\,1} - \nabla_j\,\mathcal{A}^1_{i\,1}, \tag{2.44}$$

$$\mathcal{F}^p_{kij} = \nabla_i\,\mathcal{A}^p_{j\,k} - \nabla_j\,\mathcal{A}^p_{i\,k} - \frac{i\,e\,g_2}{\hbar\,c}\sum_{h=1}^{2}\left(\mathcal{A}^p_{i\,h}\,\mathcal{A}^h_{j\,k} - \mathcal{A}^p_{j\,h}\,\mathcal{A}^h_{i\,k}\right), \tag{2.45}$$

$$\mathcal{F}^p_{kij} = \nabla_i\,\mathcal{A}^p_{j\,k} - \nabla_j\,\mathcal{A}^p_{i\,k} - \frac{i\,e\,g_3}{\hbar\,c}\sum_{h=1}^{3}\left(\mathcal{A}^p_{i\,h}\,\mathcal{A}^h_{j\,k} - \mathcal{A}^p_{j\,h}\,\mathcal{A}^h_{i\,k}\right). \tag{2.46}$$

Due to (2.44), (2.45), and (2.46) the formulas (2.40), (2.41), (2.42) are written as

$$\mathfrak{R}^1_{1ij} = -\frac{i\,e\,g_1}{\hbar\,c}\,\mathcal{F}_{ij}, \qquad \mathfrak{R}^p_{kij} = -\frac{i\,e\,g_2}{\hbar\,c}\,\mathcal{F}^p_{kij}, \qquad \mathfrak{R}^p_{kij} = -\frac{i\,e\,g_3}{\hbar\,c}\,\mathcal{F}^p_{kij}, \tag{2.47}$$

The concordance conditions (2.8), (2.9), and (2.10) should be fulfilled for both vacuum and non-vacuum connections in (2.36), (2.37), and (2.38). Therefore, from the formulas (2.11), (2.12), and (2.13) we derive

$$\mathcal{A}^1_{k1} = \overline{\mathcal{A}^1_{k1}}, \tag{2.48}$$

$$\sum_{a=1}^{2} D_{a\bar{j}}\,\mathcal{A}^a_{ki} = \sum_{\bar{a}=1}^{2} D_{i\bar{a}}\,\overline{\mathcal{A}^{\bar{a}}_{k\bar{j}}}, \tag{2.49}$$

$$\sum_{a=1}^{2} \mathcal{A}^a_{ki}\,d_{aj} = \sum_{a=1}^{2} \mathcal{A}^a_{kj}\,d_{ai}. \tag{2.50}$$

Similarly, from the formulas (2.14) and (2.15) we derive

$$\sum_{a=1}^{3} D_{a\bar{j}}\,\mathcal{A}^a_{ki} = \sum_{\bar{a}=1}^{3} D_{i\bar{a}}\,\overline{\mathcal{A}^{\bar{a}}_{k\bar{j}}}, \tag{2.51}$$

$$\sum_{a=1}^{3} d_{ajm}\,\mathcal{A}^a_{ki} + \sum_{a=1}^{3} d_{iam}\,\mathcal{A}^a_{kj} + \sum_{a=1}^{3} d_{ija}\,\mathcal{A}^a_{km} = 0. \tag{2.52}$$

Now let's substitute (2.47) into (2.21), (2.22), (2.23), (2.24), and (2.25). As a result we get a series of formulas similar to (2.48), (2.49), (2.50), (2.51), and (2.52):

$$\mathcal{F}_{ij}=\overline{\mathcal{F}_{ij}},\tag{2.53}$$

$$\sum_{\bar{k}=1}^{2}\mathcal{F}^{k}_{pij}\,\mathfrak{D}_{k\bar{q}}=\sum_{k=1}^{2}\overline{\mathcal{F}^{\bar{k}}_{\bar{q}ij}}\,\mathfrak{D}_{p\bar{k}},\tag{2.54}$$

$$\sum_{\bar{k}=1}^{3}\boldsymbol{\mathcal{F}}^{k}_{pij}\,\boldsymbol{\mathfrak{D}}_{k\bar{q}}=\sum_{k=1}^{3}\overline{\boldsymbol{\mathcal{F}}^{\bar{k}}_{\bar{q}ij}}\,\boldsymbol{\mathfrak{D}}_{p\bar{k}}.\tag{2.55}$$

$$\sum_{k=1}^{2}\mathcal{F}^{k}_{pij}\,\mathfrak{d}_{kq}=\sum_{k=1}^{2}\mathcal{F}^{k}_{qij}\,\mathfrak{d}_{kp},$$

$$\sum_{k=1}^{3}\boldsymbol{\mathfrak{d}}_{kqm}\,\boldsymbol{\mathcal{F}}^{k}_{pij}+\sum_{k=1}^{3}\boldsymbol{\mathfrak{d}}_{pkm}\,\boldsymbol{\mathcal{F}}^{k}_{qij}+\sum_{k=1}^{3}\boldsymbol{\mathfrak{d}}_{pqk}\,\boldsymbol{\mathcal{F}}^{k}_{qij}=0.$$

Substituting (2.47) into (2.29), (2.30), and (2.31), we derive the following identities:

$$D^{11}\,D_{11}\,\mathcal{F}_{ij}\,\overline{\mathcal{F}_{mn}}=\mathcal{F}_{ij}\,\mathcal{F}_{mn},\tag{2.56}$$

$$\sum_{p=1}^{2}\sum_{\bar{p}=1}^{2}\sum_{q=1}^{2}\sum_{\bar{q}=1}^{2}\mathfrak{D}^{q\bar{q}}\,\mathfrak{D}_{p\bar{p}}\,\mathcal{F}^{p}_{qij}\,\overline{\mathcal{F}^{\bar{p}}_{\bar{q}mn}}=\sum_{p=1}^{2}\sum_{q=1}^{2}\mathcal{F}^{p}_{qij}\,\mathcal{F}^{q}_{pij},\tag{2.57}$$

$$\sum_{p=1}^{3}\sum_{\bar{p}=1}^{3}\sum_{q=1}^{3}\sum_{\bar{q}=1}^{3}\boldsymbol{\mathfrak{D}}^{q\bar{q}}\,\boldsymbol{\mathfrak{D}}_{p\bar{p}}\,\boldsymbol{\mathcal{F}}^{p}_{qij}\,\overline{\boldsymbol{\mathcal{F}}^{\bar{p}}_{\bar{q}mn}}=\sum_{p=1}^{3}\sum_{q=1}^{3}\boldsymbol{\mathcal{F}}^{p}_{qij}\,\boldsymbol{\mathcal{F}}^{q}_{pij}.\tag{2.58}$$

The identities (2.56), (2.57), (2.58) can be derived directly from (2.53), (2.54), (2.55) with the use of the formulas (2.32), (2.33), and (2.34).

Let's denote by $\mathcal{L}_1=\mathcal{L}_1(\mathbf{F})$, $\mathcal{L}_2=\mathcal{L}_2(\mathbf{F})$, and $\mathcal{L}_3=\mathcal{L}_3(\mathbf{F})$ the kinetic terms of the gauge fields in the action integral of the Standard Model:

$$\mathcal{L}_1=-\frac{1}{16\,\pi\,c}\int\sum_{i,j,m,n=0}^{3}\!\!\ldots\!\!\sum g^{im}\,g^{jn}\,\mathcal{F}^{1}_{1ij}\,\mathcal{F}^{1}_{1mn}\,dV,\tag{2.59}$$

$$\mathcal{L}_2=-\frac{1}{32\,\pi\,c}\int\sum_{p=1}^{2}\sum_{q=1}^{2}\sum_{i,j,m,n=0}^{3}\!\!\ldots\!\!\sum g^{im}\,g^{jn}\,\mathcal{F}^{p}_{qij}\,\mathcal{F}^{q}_{pij}\,dV,\tag{2.60}$$

$$\mathcal{L}_3=-\frac{1}{48\,\pi\,c}\int\sum_{p=1}^{3}\sum_{q=1}^{3}\sum_{i,j,m,n=0}^{3}\!\!\ldots\!\!\sum g^{im}\,g^{jn}\,\boldsymbol{\mathcal{F}}^{p}_{qij}\,\boldsymbol{\mathcal{F}}^{q}_{pij}\,dV.\tag{2.61}$$

Here dV is the 4-dimensional volume element in the base space-time manifold M:

$$dV=\sqrt{-\det\mathbf{g}}\;d^4x=\sqrt{-\det\mathbf{g}}\;\,dx^0\wedge dx^1\wedge dx^2\wedge dx^3.\tag{2.62}$$

Due to the identities (2.56), (2.57), (2.58) the integrals (2.59), (2.60), and (2.61) are real numbers. The coefficients preceding these integrals in (2.59), (2.60), and (2.61) are chosen by analogy to the Electrodynamics (see [8]).

Definition 2.3. A Higgs field (2.1) is called a flat classical Higgs vacuum if

$$\sum_{\alpha=1}^{2}\sum_{\bar{\alpha}=1}^{2} \mathit{D}_{\alpha\bar{\alpha}}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \varphi^{\alpha 111}\, \overline{\varphi^{\bar{\alpha}111}} = \text{const}, \qquad \nabla_k \varphi^{\alpha 111} = 0, \tag{2.63}$$

where the covariant derivatives ∇_k (see (2.2)) are determined by the flat connections forming a classical vacuum of gauge fields in the sense of the definition 2.2.

Due to (2.8) and (2.9) the conditions (2.63) are consistent with each other. Note that the Higgs field $\varphi^{\alpha 111}$ has neither spinor nor tensorial indices associated with the tangent bundle TM. It is a scalar field with respect to Lorentz transformations. For this reason the flatness condition (2.35) fulfilled for the vacuum gauge fields guarantees the existence of local nonzero Higgs vacua. As for a global nonzero Higgs vacuum, its existence depends on the topology of the base manifold M and the electro-weak bundles UM and SUM over this base. Below we implicitly assume that at least one nonzero classical Higgs vacuum does exist. We denote it by $\boldsymbol{\varphi}[vac]$.

For any given Higgs field (2.1) we denote by $|\boldsymbol{\varphi}|^2$ the left hand side of the first formula (2.63) in the above definition 2.3:

$$|\boldsymbol{\varphi}|^2 = \sum_{\alpha=1}^{2}\sum_{\bar{\alpha}=1}^{2} \mathit{D}_{\alpha\bar{\alpha}}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \varphi^{\alpha 111}\, \overline{\varphi^{\bar{\alpha}111}}. \tag{2.64}$$

In terms of (2.64) the potential of the Higgs field is written as

$$V(\boldsymbol{\varphi}) = \lambda\, |\boldsymbol{\varphi}|^4 - \mu^2\, |\boldsymbol{\varphi}|^2 \tag{2.65}$$

(see [3]). Here λ and μ are two constants. They are parameters of the Standard Model. By analogy to (2.64) we denote by $|\nabla\boldsymbol{\varphi}|^2$ the following expression:

$$|\nabla\boldsymbol{\varphi}|^2 = \sum_{i=0}^{3}\sum_{j=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar{\alpha}=1}^{2} g^{ij}\, \mathit{D}_{\alpha\bar{\alpha}}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \mathit{D}_{11}\, \nabla_i\, \varphi^{\alpha 111}\, \overline{\nabla_j\, \varphi^{\bar{\alpha}111}}. \tag{2.66}$$

The expressions (2.65) and (2.66) determine the potential and kinetic terms for the Higgs field in the action integral of the Standard Model:

$$\mathcal{L}_4 = \frac{\hbar^2}{2\, m_\varphi\, c} \int |\nabla\boldsymbol{\varphi}|^2\, dV, \tag{2.67}$$

$$\mathcal{L}_5 = -\frac{m_\varphi\, c}{2} \int V(\boldsymbol{\varphi})\, dV. \tag{2.68}$$

Here $|\nabla\boldsymbol{\varphi}|^2$ and $V(\boldsymbol{\varphi})$ are given by the formulas (2.66) and (2.65), while dV is the 4-dimensional volume element given by the formula (2.62). Like λ amd μ, the constant m_φ is a parameter of the Standard Model, it is interpreted as the mass of the Higgs field. Let's take the restricted action

$$\mathcal{L}_\varphi = \mathcal{L}_4 + \mathcal{L}_5. \tag{2.69}$$

The extremum of the restricted action (2.69) is determined by the following Klein-Gordon-Fock type equation for the scalar Higgs field $\boldsymbol{\varphi}$:

$$-\frac{\hbar^2}{2\,m_\chi\,c}\sum_{i=0}^{3}\sum_{j=0}^{3}g^{ij}\,\nabla_i\nabla_j\,\varphi^{\alpha 111}+\frac{m_\chi\,c}{2}\left(2\,\lambda\,|\boldsymbol{\varphi}|^2\,-\mu^2\right)\varphi^{\alpha 111}=0. \tag{2.70}$$

The equation (2.70) has the trivial symmetric solution

$$\boldsymbol{\varphi}=0. \tag{2.71}$$

However, apart from this trivial solution (2.71), we shall consider a nontrivial global solution described by the above definition 2.3:

$$\boldsymbol{\varphi}=\boldsymbol{\varphi}[vac]. \tag{2.72}$$

Its existence is an assumption of the Standard Model when it is implemented on a non-flat space-time manifold M in General Relativity. Applying (2.63) to the equation (2.70), for the nontrivial Higgs vacuum we find

$$|\boldsymbol{\varphi}[vac]|^2=\frac{\mu^2}{2\,\lambda}=\operatorname{const}. \tag{2.73}$$

The constant (2.73) is denoted be $v^2/2$. Then we can express λ through μ and v:

$$|\boldsymbol{\varphi}[vac]|^2=\frac{v^2}{2}, \qquad\qquad \lambda=\frac{\mu^2}{v^2}. \tag{2.74}$$

Having defined the gauge and Higgs vacuum fields, now we proceed to other matter fields, i. e. to leptons and quarks. Vacuum values of their fields are trivial:

$$\boldsymbol{\psi}[e][vac]=0, \qquad \boldsymbol{\psi}[\mu][vac]=0, \qquad \boldsymbol{\psi}[\tau][vac]=0, \tag{2.75}$$

$$\boldsymbol{\psi}[1][vac]=0, \qquad \boldsymbol{\psi}[2][vac]=0, \qquad \boldsymbol{\psi}[3][vac]=0 \tag{2.76}$$

(compare (2.75) and (2.76) with (1.3) and (1.12)). The formulas (2.75) and (2.76) mean that both chiral and antichiral components of vacuum fields (1.4), (1.5), (1.13), and (1.14) are zero.

Gauge transformations and perturbations of the Higgs vacuum.

Unlike the trivial Higgs vacuum (2.71), a nontrivial vacuum (2.72) is not unique. Applying some gauge transformation to a given one, we can get another nontrivial Higgs vacuum equivalent to it. Assume that some orthonormal frame (U, $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$) of the bundle SUM is fixed (see definition in [2]). Then the basic fields of this bundle $\mathbf{D}$ and $\mathbf{d}$ are given by the following constant matrices:

$$D_{i\bar{j}}=\begin{Vmatrix}1&0\\0&1\end{Vmatrix}, \qquad\qquad d_{ij}=\begin{Vmatrix}0&1\\-1&0\end{Vmatrix}. \tag{3.1}$$

The concordance conditions (2.12) and (2.13) written for (3.1) then mean that for each fixed k the connection components A^a_{ki} form a matrix from the Lie algebra su(2) of the special unitary group SU(2). This fact reflects the SU(2) symmetry of the Standard Model. Now assume that $\Omega^\alpha_\beta(p)$ is a matrix-valued smooth function with the argument $p\in M$ and with values $\Omega^\alpha_\beta(p)\in\mathrm{SU}(2)$. Let's denote $[\Omega^{-1}(p)]^\alpha_\beta$ the components of the inverse matrix for $\Omega^\alpha_\beta(p)$. Then for each fixed k the following quantities form a matrix from the Lie algebra su(2):

$$\omega^a_{ki}=\sum_{s=1}^{2}L_{\boldsymbol{\Upsilon}_k}(\Omega^a_s(p))\,[\Omega^{-1}(p)]^s_i. \tag{3.2}$$

The SU(2) gauge transformation for the Higgs field $\boldsymbol{\varphi}$ is given by the formulas

$$\varphi^{\alpha 111}\longmapsto\sum_{\beta=1}^{2}\Omega^\alpha_\beta\,\varphi^{\beta 111}, \tag{3.3}$$

$$\mathrm{A}^a_{ki}\longmapsto\sum_{s=1}^{2}\sum_{b=1}^{2}\Omega^a_b(p)\,\mathrm{A}^b_{ks}\,[\Omega^{-1}(p)]^s_i-\omega^a_{ki}, \tag{3.4}$$

where ω^a_{ki} is expressed through Ω^α_β according to the formula (3.2). Using (3.3) and (3.4) and taking into account (3.2), we derive

$$\nabla_k\varphi^{\alpha 111}\longmapsto\sum_{\beta=1}^{2}\Omega^\alpha_\beta\,\nabla_k\varphi^{\beta 111}, \tag{3.5}$$

$$\nabla_i\nabla_j\,\varphi^{\alpha 111}\longmapsto\sum_{\beta=1}^{2}\Omega^\alpha_\beta\,\nabla_i\nabla_j\,\varphi^{\alpha 111}. \tag{3.6}$$

Due to (3.5) and (3.6), applying a gauge transformation (3.3) to some solution of the equation (2.70), we get a solution of the similar equation with respect to the transformed connection (3.4). It is important to note that the initial and transformed connections in (3.4) have the equivalent curvature tensors, i. e. we can complement the formulas (3.3) and (3.4) with the formula

$$\mathfrak{R}^q_{kij}\longmapsto\sum_{r=1}^{2}\sum_{s=1}^{2}\Omega^q_r(p)\,\mathfrak{R}^r_{sij}\,[\Omega^{-1}(p)]^s_k. \tag{3.7}$$

Due to the formula (3.7), if the initial gauge vacuum in (2.37) is flat, then the transformed gauge vacuum obtained by means of the formula (3.4) is also flat.

Now let's consider a perturbation of the Higgs vacuum $\boldsymbol{\varphi}[vac]$. It is a tensorial field $\tilde{\boldsymbol{\varphi}}$ associated with the bundles UM and $S\bar{U}M$ of the same type as $\boldsymbol{\varphi}[vac]$:

$$\tilde{\varphi}^{\alpha 111}=\varphi^{\alpha 111}[vac]+\xi^{\alpha 111}. \tag{3.8}$$

Note that $|\boldsymbol{\varphi}|\neq 0$. The perturbation $\boldsymbol{\xi}$ in (3.8) is assumed to be sufficiently small so that $|\tilde{\boldsymbol{\varphi}}|\neq 0$. By analogy to (2.74) we denote

$$|\tilde{\boldsymbol{\varphi}}|^2=\frac{\tilde{v}^2}{2}\neq 0. \tag{3.9}$$

Due to (3.9) we can construct two tensor fields

$$\frac{\sqrt{2}\,\boldsymbol{\varphi}[vac]}{v},\qquad\qquad \frac{\sqrt{2}\,\tilde{\boldsymbol{\varphi}}}{\tilde{v}} \tag{3.10}$$

and for each point $p\in M$ treat their values as two vectors of the unit length in a two-dimensional Hermitian vector space (the Hermitian form of this space is given by the formula (2.64)).

Lemma 3.1. *For any two vectors* $\mathbf{v}$ *and* $\tilde{\mathbf{v}}$ *of the unit length in a Hermitian space of the dimension* 2 *or higher there is a unitary operator* $\boldsymbol{\Omega}$ *with the unit determinant* $\det\boldsymbol{\Omega}=1$ *such that* $\tilde{\mathbf{v}}=\boldsymbol{\Omega}(\mathbf{v})$.

This lemma is a rather simple fact from the linear algebra. Its proof is left to the reader. Applying this lemma to the vectors (3.10), we find that there is a special unitary matrix $\Omega\in\mathrm{SU}(2)$ and a gauge transformation (3.3) such that

$$\boldsymbol{\varphi}[vac]\longmapsto\frac{v}{\tilde{v}}\,\tilde{\boldsymbol{\varphi}}. \tag{3.11}$$

The formula (3.11) means that each sufficiently small perturbation of the Higgs vacuum is decomposed into elongation and rotation parts and the rotation part is gauge equivalent to the initial vacuum. For this reason only elongation type perturbations of the Higgs vacuum are considered in the Standard Model (see [3]):

$$\boldsymbol{\varphi}=\boldsymbol{\varphi}[vac]+\frac{\chi}{v}\,\boldsymbol{\varphi}[vac]. \tag{3.12}$$

Here χ is a real scalar field. It is also called the *Higgs field*. This terminology makes no confusion since both Higgs fields are closely related through the formula (3.12).

4. Symmetry breaking and boson fields recalculation.

From now on assume that some nontrivial Higgs vacuum is fixed. On fixing it we say that the initial $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry is broken. However, some definite amount of this symmetry remains unbroken. Remember that $\nabla\boldsymbol{\varphi}=0$ with respect to the vacuum connections in (2.36) and (2.37). Let's study those gauge fields $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{A}}$ in (2.36) and (2.37) that preserve this equality:

$$\nabla_k\,\varphi^{\alpha111}[vac]-\frac{i\,e}{\hbar\,c}\left(\sum_{\theta=1}^{2}g_2\,\mathcal{A}^{\alpha}_{k\theta}\,\varphi^{\theta111}[vac]+3\,g_1\,\mathcal{A}^{1}_{k1}\,\varphi^{\alpha111}[vac]\right)=0. \tag{4.1}$$

Since $\nabla_k\,\varphi^{\alpha111}[vac]=0$, the equality (4.1) reduces to a non-differential equality:

$$\sum_{\theta=1}^{2}g_2\,\mathcal{A}^{\alpha}_{k\theta}\,\varphi^{\theta111}[vac]+3\,g_1\,\mathcal{A}^{1}_{k1}\,\varphi^{\alpha111}[vac]=0. \tag{4.2}$$

This equality (4.2) here should be treated as a complement to the concordance conditions (2.48), (2.49), and (2.50). In order to solve the total set of the equations (4.2), (2.48), (2.49), (2.50) we introduce a projector $\mathbf{P}[vac]$ with the components:

$$P^{\beta}_{\alpha}[vac]=\sum_{\bar{\alpha}=1}^{2}\frac{\mathit{Đ}_{\alpha\bar{\alpha}}\,\mathit{Đ}_{11}\,\mathit{Đ}_{11}\,\mathit{Đ}_{11}}{|\boldsymbol{\varphi}[vac]|^2}\,\overline{\varphi^{\bar{\alpha}111}[vac]}\,\varphi^{\beta111}[vac]. \tag{4.3}$$

The projection operator with the components (4.3) is an orthogonal projector since it is a Hermitian operator in the sense of the following equality:

$$\sum_{a=1}^{2} \text{Đ}_{a\bar{j}}\, P^{a}_{i}[vac] = \sum_{\bar{a}=1}^{2} \text{Đ}_{i\bar{a}}\, \overline{P^{\bar{a}}_{\bar{j}}[vac]}. \tag{4.4}$$

Now, in addition to $\boldsymbol{\varphi}[vac]$, we introduce another field $\boldsymbol{\phi}[vac]$ derived from it. This field can also be called the Higgs vacuum field:

$$\phi^{\alpha}_{111}[vac] = \sum_{\beta=1}^{2}\sum_{\bar{\beta}=1}^{2} \text{đ}^{\alpha\beta}\, \text{Đ}_{\beta\bar{\beta}}\, \text{Đ}_{11}\, \text{Đ}_{11}\, \text{Đ}_{11}\, \overline{\varphi^{\bar{\beta}111}[vac]}. \tag{4.5}$$

The modulus of the field (4.5) is determined by the formula similar to (2.64):

$$|\boldsymbol{\phi}|^2 = \sum_{\alpha=1}^{2}\sum_{\bar{\alpha}=1}^{2} \text{Đ}_{\alpha\bar{\alpha}}\, D^{11}\, D^{11}\, D^{11}\, \phi^{\alpha}_{111}\, \overline{\phi^{\bar{\alpha}}_{111}}. \tag{4.6}$$

Using (2.16), (2.32), and (2.33), from (2.64), (2.73), and (4.6) one easily derives

$$|\boldsymbol{\phi}[vac]|^2 = |\boldsymbol{\varphi}[vac]|^2 = \frac{v^2}{2} = \text{const}\,. \tag{4.7}$$

Due to (4.7), using $\boldsymbol{\phi}[vac]$, one can define another projection operator $\mathbf{Q}[vac]$:

$$Q^{\beta}_{\alpha}[vac] = \sum_{\bar{\alpha}=1}^{2} \frac{\text{Đ}_{\alpha\bar{\alpha}}\, D^{11}\, D^{11}\, D^{11}}{|\boldsymbol{\phi}[vac]|^2}\, \overline{\phi^{\bar{\alpha}}_{111}[vac]}\, \phi^{\beta}_{111}[vac]. \tag{4.8}$$

Like the projector $\mathbf{P}[vac]$ with the components (4.3), the projector $\mathbf{Q}[vac]$ witn the components (4.8) is an orthogonal projector. This fact is expressed by the equality

$$\sum_{a=1}^{2} \text{Đ}_{a\bar{j}}\, Q^{a}_{i}[vac] = \sum_{\bar{a}=1}^{2} \text{Đ}_{i\bar{a}}\, \overline{Q^{\bar{a}}_{\bar{j}}[vac]}. \tag{4.9}$$

It easy to see that tne equality (4.9) is analogous to (4.4).

Two Higgs fields $\boldsymbol{\phi}[vac]$ and $\boldsymbol{\varphi}[vac]$ are perpendicular to each other in the sense of the following equality reflecting the Hermitian forms in (2.64) and (4.6):

$$\sum_{\alpha=1}^{2}\sum_{\bar{\alpha}=1}^{2} \text{Đ}_{\alpha\bar{\alpha}}\, \overline{\phi^{\bar{\alpha}}_{111}[vac]}\, \varphi^{\alpha 111}[vac] = 0. \tag{4.10}$$

Due (4.10) the projectors $\mathbf{P}[vac]$ and $\mathbf{Q}[vac]$ are complementary to each other:

$$\mathbf{P}[vac] + \mathbf{Q}[vac] = \mathbf{id}\,. \tag{4.11}$$

In a coordinate form the operator equality (4.11) looks like

$$P^{\beta}_{\alpha}[vac] + Q^{\beta}_{\alpha}[vac] = \delta^{\beta}_{\alpha}. \tag{4.12}$$

Apart from (4.11) and (4.12), there are some other consequences of the orthogonality condition (4.10). Though $\boldsymbol{\phi}[vac]$ and $\boldsymbol{\varphi}[vac]$ are two tensorial fields of the different types, due to (4.7) and (4.10) one can treat them as a kind of orthogonal frame in the two-dimensional complex bundle SUM. Relying upon this interpretation of $\boldsymbol{\phi}[vac]$ and $\boldsymbol{\varphi}[vac]$, we introduce two tensor fields $\mathbf{W}[\varphi\blacktriangleright\phi]$ and $\mathbf{W}[\phi\blacktriangleright\varphi]$:

$$W^{\beta}_{\alpha 111111}[\varphi\blacktriangleright\phi]=\sum^{2}_{\bar{a}=1}\frac{\eth_{\alpha\bar{\alpha}}\,\eth_{11}\,\eth_{11}\,\eth_{11}}{|\boldsymbol{\varphi}[vac]|^2}\,\overline{\varphi^{\bar{\alpha}111}[vac]}\,\phi^{\beta}_{111}[vac],\tag{4.13}$$

$$W^{\beta 111111}_{\alpha}[\phi\blacktriangleright\varphi]=\sum^{2}_{\bar{a}=1}\frac{\eth_{\alpha\bar{\alpha}}\,\eth^{11}\,\eth^{11}\,\eth^{11}}{|\boldsymbol{\phi}[vac]|^2}\,\overline{\phi^{\bar{\alpha}}_{111}[vac]}\,\varphi^{\beta 111}[vac].\tag{4.14}$$

It easy to derive the following identities for the tensor fields (4.13) and (4.14):

$$\sum^{2}_{\alpha=1}W^{\beta}_{\alpha 111111}[\varphi\blacktriangleright\phi]\;W^{\alpha}_{\gamma 111111}[\varphi\blacktriangleright\phi]=0,\tag{4.15}$$

$$\sum^{2}_{\alpha=1}W^{\beta 111111}_{\alpha}[\phi\blacktriangleright\varphi]\;W^{\alpha 111111}_{\gamma}[\phi\blacktriangleright\varphi]=0,\tag{4.16}$$

$$\sum^{2}_{\alpha=1}W^{\beta}_{\alpha 111111}[\varphi\blacktriangleright\phi]\;W^{\alpha 111111}_{\gamma}[\phi\blacktriangleright\varphi]=Q^{\beta}_{\gamma}[vac],\tag{4.17}$$

$$\sum^{2}_{\alpha=1}W^{\beta 111111}_{\alpha}[\phi\blacktriangleright\varphi]\,W^{\alpha}_{\gamma 111111}[\varphi\blacktriangleright\phi]=P^{\beta}_{\gamma}[vac].\tag{4.18}$$

In a coordinate-free form the identties (4.15), (4.16), (4.17), (4.18) are written as

$$\mathbf{W}[\varphi\blacktriangleright\phi]^2=0,\qquad\qquad \mathbf{W}[\varphi\blacktriangleright\phi]\circ\mathbf{W}[\phi\blacktriangleright\varphi]=\mathbf{Q}[vac],\tag{4.19}$$

$$\mathbf{W}[\phi\blacktriangleright\varphi]^2=0,\qquad\qquad \mathbf{W}[\phi\blacktriangleright\varphi]\circ\mathbf{W}[\varphi\blacktriangleright\phi]=\mathbf{P}[vac].\tag{4.20}$$

In addition to (4.19) and (4.20), we have

$$\mathbf{W}[\varphi\blacktriangleright\phi]\circ\mathbf{P}[vac]=\mathbf{W}[\varphi\blacktriangleright\phi],\qquad \mathbf{W}[\varphi\blacktriangleright\phi]\circ\mathbf{Q}[vac]=0,\tag{4.21}$$

$$\mathbf{Q}[vac]\circ\mathbf{W}[\varphi\blacktriangleright\phi]=\mathbf{W}[\varphi\blacktriangleright\phi],\qquad \mathbf{P}[vac]\circ\mathbf{W}[\varphi\blacktriangleright\phi]=0,\tag{4.22}$$

$$\mathbf{W}[\phi\blacktriangleright\varphi]\circ\mathbf{Q}[vac]=\mathbf{W}[\phi\blacktriangleright\varphi],\qquad \mathbf{W}[\phi\blacktriangleright\varphi]\circ\mathbf{P}[vac]=0,\tag{4.23}$$

$$\mathbf{P}[vac]\circ\mathbf{W}[\phi\blacktriangleright\varphi]=\mathbf{W}[\phi\blacktriangleright\varphi],\qquad \mathbf{Q}[vac]\circ\mathbf{W}[\phi\blacktriangleright\varphi]=0.\tag{4.24}$$

Let $A^{\dots\,\beta\,\dots}_{\dots\,\alpha\,\dots}$ be the components of some arbitrary tensorial field $\mathbf{A}$ with at least one upper index and at least one lower index associated with the two-dimensional bundle SUM. By dots we denote other indices of $\mathbf{A}$ that could be present. In this case we have the following expansion for the tensor field $\mathbf{A}$:

$$\begin{aligned}A^{\dots\,\beta\,\dots}_{\dots\,\alpha\,\dots}&=A^{+\dots}_{\dots}\cdot P^{\beta}_{\alpha}[vac]+A^{-\dots}_{\dots}\cdot Q^{\beta}_{\alpha}[vac]+\\&+W^{+111111\,\dots}_{\dots\dots\dots\dots}\cdot W^{\beta}_{\alpha 111111}[\varphi\blacktriangleright\phi]+W^{-\,\dots\dots\dots}_{111111\,\dots}\cdot W^{\beta 111111}_{\alpha}[\phi\blacktriangleright\varphi].\end{aligned}\tag{4.25}$$

The expansion (4.25) is derived with the use of the formula (4.11) and the above identities (4.21), (4.22), (4.23), (4.24). Let's apply this expansion to the components of the tensor $\boldsymbol{\mathcal{A}}$ in (2.37). As a result we get

$$\begin{gathered}\mathcal{A}^{\beta}_{k\alpha}=A^{+}_{k}\cdot P^{\beta}_{\alpha}[vac]+A^{-}_{k}\cdot Q^{\beta}_{\alpha}[vac]+\\+W^{+111111}_{k}\cdot W^{\beta}_{\alpha 111111}[\varphi\blacktriangleright\phi]+W^{-}_{k111111}\cdot W^{\beta 111111}_{\alpha}[\phi\blacktriangleright\varphi].\end{gathered}\tag{4.26}$$

Here A^{+}_{k} and A^{-}_{k} are the components of two fields which are SU(2)-singlets. Applying (2.49) and (2.50) to (4.26), we find that

$$A^{+}_{k}=\overline{A^{+}_{k}},\qquad A^{-}_{k}=\overline{A^{-}_{k}},\qquad A^{-}_{k}=-A^{+}_{k}.\tag{4.27}$$

In other words A^{+}_{k} and A^{-}_{k} are the components of two mutually opposite covectorial fields. The index k in (4.26) and (4.27) is a spatial index associated with the tangent bundle TM. In physical literature the following notation is a tradition (see [3]):

$$A^{+}_{k}=-A^{3}_{k},\qquad A^{-}_{k}=A^{3}_{k},\qquad A^{3}_{k}=\overline{A^{3}_{k}}.\tag{4.28}$$

The number 3 in (4.28) is not a tensorial index. In [3] and in many other books and papers it is set because A^{3}_{k} first appear as coefficients of the third Pauli matrix

$$\boldsymbol{\sigma}_3=\begin{Vmatrix}1&0\\0&-1\end{Vmatrix}.$$

For $W^{+111111}_{k}$ and $W^{-}_{k111111}$ in (4.26) from (2.49) we derive

$$\begin{aligned}W^{-}_{k111111}&=D_{11}\,D_{11}\,D_{11}\,D_{11}\,D_{11}\,D_{11}\,\overline{W^{+111111}_{k}},\\W^{+111111}_{k}&=D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\,\overline{W^{-}_{k111111}}.\end{aligned}\tag{4.29}$$

The index k in (4.29) is associated with the tangent bundle TM. For this reason $W^{+111111}_{k}$ and $W^{-}_{k111111}$ are the components of two covectorial fields $\mathbf{W}^{+}$ and $\mathbf{W}^{-}$. These fields correspond to W-bosons. They are scalar fields with respect to the bundle SUM, i. e. they are SU(2)-singlets, but they are not scalar with respect to the one-dimensional bundle UM.

Thus the relationships (2.49) and (2.50) are resolved to (4.28) and (4.29). Let's apply (4.26) to (4.2). As a result, taking into account (4.28) and (4.29), we get

$$\begin{aligned}&W^{-}_{k111111}=0, && W^{+111111}_{k}=0,\\&A^{+}_{k}=\frac{3\,g_1}{g_1}\,\mathcal{A}^{1}_{k1}, && A^{-}_{k}=-\frac{3\,g_1}{g_1}\,\mathcal{A}^{1}_{k1}.\end{aligned}\tag{4.30}$$

The formulas (4.30) show that there is a special combination of the gauge fields $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{A}}$ which, upon substituting into (2.36) and (2.37), preserves the classical Higgs vacuum. The gauge field $\boldsymbol{\mathcal{A}}$ is expressed through the gauge field $\boldsymbol{\mathcal{A}}$ in this combination. In other words this fact means that the 4-dimensional gauge symmetry $\mathrm{U}(2)\times\mathrm{U}(1)$ reduces to 1-dimensional gauge symmetry U(1). However, this new U(1) symmetry does not coincide with the initial U(1) symmetry being a component in the direct product SU(2) × U(1). This residual U(1) symmetry in the electro-weak bundles *SUM* and *UM* is interpreted as a U(1) symmetry of the electromagnetism. In order to reveal this hidden $\mathrm{U}(1)_{\mathrm{em}}$ symmetry the following two real fields $\mathbf{A}$ and $\mathbf{Z}$ are introduced in addition to $\mathbf{W}^{+}$ and $\mathbf{W}^{-}$:

$$Z_k=\frac{-g_2\,A^3_k+3\,g_1\,\mathcal{A}^1_{k1}}{\sqrt{(g_2)^2+(3\,g_1)^2}},\qquad\qquad A_k=\frac{3\,g_1\,A^3_k+g_2\,\mathcal{A}^1_{k1}}{\sqrt{(g_2)^2+(3\,g_1)^2}}.\tag{4.31}$$

The relationships (4.30) now mean that (4.2) is fulfilled provided

$$\mathbf{W}^{+}=0,\qquad\qquad \mathbf{W}^{-}=0,\qquad\qquad \mathbf{Z}=0.\tag{4.32}$$

The condition $\mathbf{A}=0$ is absent in (4.32) because the classical Higgs vacuum $\boldsymbol{\varphi}[vac]$ is invariant with respect to purely electromagnetic gauge transformations.

Using the relationships (4.31), we can express A^3_k and $\mathcal{A}^1_{k1}$ through A_k and Z_k. For the component of the gauge field $\boldsymbol{\mathcal{A}}$ we derive:

$$\mathcal{A}^1_{k1}=\frac{g_2\,A_k+3\,g_1\,Z_k}{\sqrt{(g_2)^2+(3\,g_1)^2}}.\tag{4.33}$$

In a similar way, from (4.31) for A^3_k we derive

$$A^3_k=\frac{3\,g_1\,A_k-g_2\,Z_k}{\sqrt{(g_2)^2+(3\,g_1)^2}}.\tag{4.34}$$

Then we substitute (4.34) back into (4.28) and (4.26). As a result we get

$$\begin{gathered}\mathcal{A}^{\beta}_{k\alpha}=\frac{3\,g_1\,A_k-g_2\,Z_k}{\sqrt{(g_2)^2+(3\,g_1)^2}}\cdot\bigl(Q^{\beta}_{\alpha}[vac]-P^{\beta}_{\alpha}[vac]\,\bigr)+\\ +\,W^{+111111}_{k}\cdot W^{\beta}_{\alpha111111}[\varphi\blacktriangleright\phi]+W^{-}_{k111111}\cdot W^{\beta111111}_{\alpha}[\phi\blacktriangleright\varphi].\end{gathered}\tag{4.35}$$

The next step is to substitute (4.35) into (2.45). We do it in several substeps:

$$\begin{gathered}\sum^{2}_{h=1}\mathcal{A}^{p}_{i\,h}\,\mathcal{A}^{h}_{j\,k}=A^3_i\,A^3_j\cdot\delta^p_q+\bigl(A^3_i\,W^{+111111}_j-A^3_j\,W^{+111111}_i\bigr)\,\cdot\\ \cdot\,W^{p}_{q111111}[\varphi\blacktriangleright\phi]+\bigl(A^3_j\,W^{-}_{i111111}-A^3_i\,W^{-}_{j111111}\bigr)\cdot W^{p111111}_{q}[\phi\blacktriangleright\varphi]+\\ +\,W^{+111111}_{i}\,W^{-}_{j111111}\cdot Q^p_q[vac]+W^{+111111}_{j}\,W^{-}_{i111111}\cdot P^p_q[vac].\end{gathered}\tag{4.36}$$

From (4.36) by means of alternation we immediately derive

$$\begin{gathered}\sum^{2}_{h=1}\Bigl(\mathcal{A}^{p}_{i\,h}\,\mathcal{A}^{h}_{j\,q}-\mathcal{A}^{p}_{j\,h}\,\mathcal{A}^{h}_{i\,q}\Bigr)=2\,\bigl(A^3_i\,W^{+111111}_j-A^3_j\,W^{+111111}_i\bigr)\,\cdot\\ \cdot\,W^{p}_{q111111}[\varphi\blacktriangleright\phi]+2\,\bigl(A^3_j\,W^{-}_{i111111}-A^3_i\,W^{-}_{j111111}\bigr)\cdot W^{p111111}_{q}[\phi\blacktriangleright\varphi]+\\ +\,\bigl(W^{+111111}_{i}\,W^{-}_{j111111}-W^{+111111}_{j}\,W^{-}_{i111111}\bigr)\cdot\bigl(Q^p_q[vac]-P^p_q[vac]\bigr)\,.\end{gathered}\tag{4.37}$$

For the differential part of the tensor Ŧ in the formula (2.45) we get

$$\begin{aligned}\nabla_i\,\mathcal{A}^p_{j\,q}-\nabla_j\,\mathcal{A}^p_{i\,q}=\frac{Q^p_q[vac]-P^p_q[vac]}{\sqrt{(g_2)^2+(3\,g_1)^2}}\cdot\bigl(3\,g_1\,(\nabla_i A_j-\nabla_j A_i)-\\ -\,g_2\,(\nabla_i Z_j-\nabla_j Z_i)\bigr)+(\nabla_i W^{+111111}_j-\nabla_j W^{+111111}_i)\,\cdot\\ \cdot\,W^p_{q111111}[\varphi\blacktriangleright\phi]+(\nabla_i W^-_{j111111}-\nabla_j W^-_{i111111})\cdot W^{p111111}_q[\phi\blacktriangleright\varphi].\end{aligned}\tag{4.38}$$

Now let's introduce the following notations analogous to (2.44):

$$F_{ij}=\nabla_i A_j-\nabla_j A_i,\qquad\qquad \mathcal{Z}_{ij}=\nabla_i Z_j-\nabla_j Z_i.\tag{4.39}$$

In the case of the fields $\mathbf{W}^+$ and $\mathbf{W}^-$ we need more complicated notations:

$$\begin{aligned}\mathcal{W}^{+111111}_{ij}&=\nabla_i W^{+111111}_j-\nabla_j W^{+111111}_i-\\ &-\frac{6\,i\,e\,g_1}{\hbar\,c}\,\bigl(\mathcal{A}^1_{i1}\,W^{+111111}_j-\mathcal{A}^1_{j1}\,W^{+111111}_i\bigr).\end{aligned}\tag{4.40}$$

$$\begin{aligned}\mathcal{W}^-_{ij111111}&=\nabla_i W^-_{j111111}-\nabla_j W^-_{i111111}+\\ &+\frac{6\,i\,e\,g_1}{\hbar\,c}\,\bigl(\mathcal{A}^1_{i1}\,W^-_{j111111}-\mathcal{A}^1_{j1}\,W^-_{i111111}\bigr).\end{aligned}\tag{4.41}$$

Here $\mathcal{A}^1_{i1}$ and $\mathcal{A}^1_{j1}$ are determined by the formula (4.33). Now we need to apply (4.39), (4.40), and (4.41) to (4.38), and combine (4.38) with (4.37) according to the formula (2.45). As a result we obtain

$$\begin{aligned}\mathcal{F}^p_{qij}=\bigl(Q^p_q[vac]-P^p_q[vac]\bigr)\cdot\Biggl(\frac{3\,g_1\,F_{ij}-\,g_2\,\mathcal{Z}_{ij}}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,-\\ -\frac{i\,e\,g_2}{\hbar\,c}\,\bigl(W^{+111111}_i\,W^-_{j111111}-W^{+111111}_j\,W^-_{i111111}\bigr)\Biggr)+\\ +\,W^p_{q111111}[\varphi\blacktriangleright\phi]\cdot\Biggl(\mathcal{W}^{+111111}_{ij}+\frac{2\,i\,e\,\sqrt{(g_2)^2+(3\,g_1)^2}}{\hbar\,c}\,\times\\ \times\,\bigl(Z_i\,W^{+111111}_j-Z_j\,W^{+111111}_i\bigr)\Biggr)+\\ +\,W^{p111111}_q[\phi\blacktriangleright\varphi]\cdot\Biggl(\mathcal{W}^-_{ij111111}-\frac{2\,i\,e\,\sqrt{(g_2)^2+(3\,g_1)^2}}{\hbar\,c}\,\times\\ \times\,\bigl(Z_i\,W^-_{j111111}-Z_j\,W^-_{i111111}\bigr)\Biggr).\end{aligned}\tag{4.42}$$

Thus, in (4.42) we have expressed the field Ŧ through the fields $\mathbf{W}^+$, $\mathbf{W}^-$, and $\mathbf{Z}$ corresponding to W and Z-bosons of the Standard Model and through the field $\mathbf{A}$ which is interpreted as the 4-dimensional vector-potential of the electromagnetic field. Our next goal is to express $\mathbf{Ғ}$ through $\mathbf{Z}$ and $\mathbf{A}$ using the formulas (4.33) and (2.44). Substituting (4.33) into (2.44) and taking into account (4.39), we derive

$$\mathcal{F}_{ij} = \frac{g_2\,F_{ij} + 3\,g_1\,\mathcal{Z}_{ij}}{\sqrt{(g_2)^2 + (3\,g_1)^2}}\,. \tag{4.43}$$

Then we substitute (4.43) and (4.42) into (2.59) and (2.60) respectively. As a result for the sum of two integrals (2.59) and (2.60) we get

$$\mathcal{L}_1 + \mathcal{L}_2 = \mathcal{L}_{11} + \mathcal{L}_{12} + \mathcal{L}_{21} + \mathcal{L}_{22} + \mathcal{L}_{23} + \mathcal{L}_{24} + \mathcal{L}_{25} + \mathcal{L}_{26}, \tag{4.44}$$

where $\mathcal{L}_{11}$ and $\mathcal{L}_{12}$ are standard kinetic terms for two real vector fields $\mathbf{A}$ and $\mathbf{Z}$:

$$\mathcal{L}_{11} = -\frac{1}{16\,\pi\,c}\int\sum_{i=0}^{3}\sum_{j=0}^{3} F_{ij}\,F^{ij}\,dV, \tag{4.45}$$

$$\mathcal{L}_{12} = -\frac{1}{16\,\pi\,c}\int\sum_{i=0}^{3}\sum_{j=0}^{3} \mathcal{Z}_{ij}\,\mathcal{Z}^{ij}\,dV. \tag{4.46}$$

The term $\mathcal{L}_{21}$ in (4.44) is a standard kinetic term for two mutually conjugate complex vector fields $\mathbf{W}^{+}$ and $\mathbf{W}^{-}$ (see (4.29)):

$$\mathcal{L}_{21} = -\frac{1}{16\,\pi\,c}\int\sum_{i=0}^{3}\sum_{j=0}^{3} \mathcal{W}^{+111111}_{ij}\,\mathcal{W}^{-ij}_{111111}\,dV. \tag{4.47}$$

The term $\mathcal{L}_{22}$ is a purely potential term responsible for the self-action of W-bosons:

$$\begin{aligned}\mathcal{L}_{22} = &-\frac{\left(\dfrac{e\,g_2}{\hbar\,c}\right)^2}{8\,\pi\,c}\int\sum_{i=0}^{3}\sum_{j=0}^{3} W^{+111111}_{i}\,W^{-i}_{111111}\,W^{+111111}_{j}\,W^{-j}_{111111}\,dV\,+\\ &+\frac{\left(\dfrac{e\,g_2}{\hbar\,c}\right)^2}{8\,\pi\,c}\int\sum_{i=0}^{3}\sum_{j=0}^{3} W^{+111111}_{i}\,W^{+i111111}\,W^{-}_{j111111}\,W^{-j}_{111111}\,dV.\end{aligned} \tag{4.48}$$

The term $\mathcal{L}_{23}$ is a mixed term responsible the interaction of W-bosons and photons:

$$\mathcal{L}_{23} = \frac{\dfrac{3\,i\,e\,g_2\,g_1}{8\,\pi\,\hbar\,c^2}}{\sqrt{(g_2)^2 + (3\,g_1)^2}}\int\sum_{i=0}^{3}\sum_{j=0}^{3} F^{ij}\,W^{+111111}_{i}\,W^{-}_{j111111}\,dV. \tag{4.49}$$

The term $\mathcal{L}_{24}$ is a mixed term responsible the interaction of W and Z-bosons:

$$\mathcal{L}_{24} = -\frac{\dfrac{i\,e\,(g_2)^2}{8\,\pi\,\hbar\,c^2}}{\sqrt{(g_2)^2 + (3\,g_1)^2}}\int\sum_{i=0}^{3}\sum_{j=0}^{3} \mathcal{Z}^{ij}\,W^{+111111}_{i}\,W^{-}_{j111111}\,dV. \tag{4.50}$$

The term $\mathcal{L}_{25}$ is a potential term responsible for the interaction of W and Z-bosons:

$$\begin{aligned}\mathcal{L}_{25}=\frac{e^2((g_2)^2+(3\,g_1)^2)}{2\,\pi\,\hbar^2\,c^3}\int\sum_{i=0}^{3}\sum_{j=0}^{3}Z_i\,Z^i\,W^{+111111}_j\,W^{-j}_{111111}\,dV-\\-\frac{e^2((g_2)^2+(3\,g_1)^2)}{2\,\pi\,\hbar^2\,c^3}\int\sum_{i=0}^{3}\sum_{j=0}^{3}Z^i\,W^{+111111}_i\,Z_j\,W^{-j}_{111111}\,dV.\end{aligned}\tag{4.51}$$

The term $\mathcal{L}_{26}$ is a mixed term responsible the interaction of W and Z-bosons:

$$\begin{aligned}\mathcal{L}_{25}=\frac{i\,e\sqrt{(g_2)^2+(3\,g_1)^2}}{4\,\pi\,\hbar\,c^2}\int\sum_{i=0}^{3}\sum_{j=0}^{3}\mathcal{W}^{+111111}_{ij}\,Z^i\,W^{-j}_{111111}\,dV-\\-\frac{i\,e\sqrt{(g_2)^2+(3\,g_1)^2}}{4\,\pi\,\hbar\,c^2}\int\sum_{i=0}^{3}\sum_{j=0}^{3}\mathcal{W}^{-ij}_{111111}\,Z_i\,W^{+111111}_j\,dV.\end{aligned}\tag{4.52}$$

The terms (4.49), (4.50), (4.51), and (4.52) are cubic with respect to fields. The terms (4.48) is the fourth order term. None of these terms can be treated as a mass term corresponding to the kinetic terms (4.45), (4.46), or (4.46).

5. Masses of boson fields.

In order to find the massive terms corresponding to the kinetic terms (4.45), (4.46), and (4.47) we consider again the elongation type perturbation of the Higgs vacuum (3.12). In a coordinate form it is given by the formula

$$\varphi^{\alpha 111}=\varphi^{\alpha 111}[vac]+\frac{\chi}{v}\,\varphi^{\alpha 111}[vac].\tag{5.1}$$

Having non-vacuum Higgs field (5.1), we should apply non-vacuum covariant differentiation to it in (2.67), i. e. we should use non-vacuum connections (2.36) and (2.37) instead of purely vacuum ones. Then we get

$$\begin{aligned}\nabla_k\varphi^{\alpha 111}=-\frac{i\,e}{\hbar\,c}\left(\sum_{\theta=1}^{2}g_2\,\mathcal{A}^{\alpha}_{k\theta}\,\varphi^{\theta 111}[vac]+3\,g_1\,\mathcal{A}^{1}_{k1}\,\varphi^{\alpha 111}[vac]\right)\times\\\times\left(1+\frac{\chi}{v}\right)+\frac{\nabla_k\chi}{v}\,\varphi^{\alpha 111}[vac].\end{aligned}\tag{5.2}$$

Note that ∇_k in the left hand side of (5.2) is a non-vacuum nabla, while $\nabla_k\chi$ is a vacuum covariant derivative applied to the real scalar Higgs field χ. Let's apply (4.26) to $\mathcal{A}^{\alpha}_{k\theta}$ in (5.2). As a result we derive

$$\begin{aligned}\nabla_k\varphi^{\alpha 111}=-\frac{i\,e}{\hbar\,c}\left(g_2\,A^{+}_k+3\,g_1\,\mathcal{A}^{1}_{k1}\right)\left(1+\frac{\chi}{v}\right)\varphi^{\alpha 111}[vac]-\\-\frac{i\,e}{\hbar\,c}\;g_2\,W^{+111111}_k\left(1+\frac{\chi}{v}\right)\phi^{\alpha}_{111}[vac]+\frac{\nabla_k\chi}{v}\,\varphi^{\alpha 111}[vac].\end{aligned}\tag{5.3}$$

Now remember the formulas (4.28) and (4.31). Then (5.3) is rewritten as

$$\begin{aligned}\nabla_k\varphi^{\alpha 111} &= -\frac{i\,e}{\hbar\,c}\,\sqrt{(g_2)^2+(3\,g_1)^2}\;Z_k\left(1+\frac{\chi}{v}\right)\varphi^{\alpha 111}[vac]-\\&-\frac{i\,e}{\hbar\,c}\,g_2\,W_k^{+111111}\left(1+\frac{\chi}{v}\right)\phi^{\alpha}_{111}[vac]+\frac{\nabla_k\chi}{v}\,\varphi^{\alpha 111}[vac].\end{aligned}\tag{5.4}$$

Let's substitute (5.4) into the formula (2.66) and take into account (4.7) and the orthogonality condition (4.10). This yields

$$\begin{aligned}|\nabla\boldsymbol{\varphi}|^2 &= \frac{e^2\,((g_2)^2+(3\,g_1)^2)}{\hbar^2\,c^2}\,\frac{(v+\chi)^2}{2}\sum_{i=0}^{3}Z_i\,Z^i+\\&+\frac{1}{2}\sum_{i=0}^{3}\sum_{j=0}^{3}g^{ij}\,\nabla_i\,\chi\,\nabla_j\,\chi+\frac{e^2\,(g_2)^2}{\hbar^2\,c^2}\,\frac{(v+\chi)^2}{2}\,\sum_{i=0}^{3}W_i^{+111111}\,W^{-i}_{111111}.\end{aligned}\tag{5.5}$$

The next step is to substitute (5.1) into (2.64). As a result we get

$$|\boldsymbol{\varphi}|^2=\frac{(v+\chi)^2}{2}.\tag{5.6}$$

Then we substitute (5.6) into (2.65) and derive

$$V(\boldsymbol{\varphi})=\lambda\,\frac{(v+\chi)^4}{4}-\mu^2\,\frac{(v+\chi)^2}{2}.\tag{5.7}$$

It is clear that $V(\boldsymbol{\varphi})$ in (5.7) is a fourth order polynomial with respect to χ:

$$\begin{aligned}V(\boldsymbol{\varphi}) &= \frac{\lambda}{4}\,\chi^4+\lambda\,v\,\chi^3+\left(\frac{3\,\lambda}{2}\,v^2-\frac{\mu^2}{2}\right)\,\chi^2+\\&+\bigl(\lambda\,v^3-\mu^2\,v\bigr)\;\chi+\left(\frac{\lambda}{4}\,v^4-\frac{\mu^2}{2}\,v^2\right).\end{aligned}\tag{5.8}$$

Due to the second equality (2.74) the term linear in χ in the above polynomial (5.8) does vanish. The polynomial $V(\boldsymbol{\varphi})$ simplifies to

$$V(\boldsymbol{\varphi})=\frac{\lambda}{4}\,\chi^4+\lambda\,v\,\chi^3+\mu^2\,\chi^2-\frac{\mu^2}{4}\,v^2.\tag{5.9}$$

Before substituting (5.5) and (5.9) into (2.67) and (2.68), let's denote

$$m_\chi=2\,m_\varphi.\tag{5.10}$$

Then, substituting (5.5) and (5.9) into (2.67) and (2.68) and using (5.10), we get

$$\mathcal{L}_4+\mathcal{L}_5=\mathcal{L}_{41}+\mathcal{L}_{42}+\mathcal{L}_{43}+\mathcal{L}_{44}+\mathcal{L}_{45}+\mathcal{L}_{51}+\mathcal{L}_{52}+\mathcal{L}_{53}.\tag{5.11}$$

The terms $\mathcal{L}_{41}$ and $\mathcal{L}_{51}$ in the sum (5.11) are the kinetic term and the mass term of the real scalar Higgs field χ respectively. They are given by the formulas

$$\mathcal{L}_{41} = \frac{\hbar^2}{2\, m_\chi\, c} \int \sum_{i=0}^{3} \sum_{j=0}^{3} g^{ij}\, \nabla_i\, \chi\; \nabla_j\, \chi\, dV, \tag{5.12}$$

$$\mathcal{L}_{51} = -\frac{m_\chi\, c}{2} \int \chi^2\, dV. \tag{5.13}$$

In order to fit (5.12) and (5.13) the parameter μ in (5.9) should be chosen so that

$$\mu^2 = 2.$$

The term $\mathcal{L}_{42}$ in (5.11) is the mass term corresponding to the kinetic term (4.46):

$$\mathcal{L}_{42} = \frac{c\, m_Z^2}{8\, \pi\, \hbar^2} \int \sum_{i=0}^{3} Z_i\, Z^i\, dV. \tag{5.14}$$

Comparing (5.14) and (5.5), we derive the formula for the mass of Z-bosons:

$$m_Z = \sqrt{\frac{4\, \pi\, ((g_2)^2 + (3\, g_1)^2)}{m_\chi}\, \frac{e\, v\, \hbar}{c^2}}. \tag{5.15}$$

The term $\mathcal{L}_{43}$ in (5.11) is the mass term corresponding to the kinetic term (4.47):

$$\mathcal{L}_{43} = \frac{c\, m_W^2}{8\, \pi\, \hbar^2} \int \sum_{i=0}^{3} W_i^{+111111}\, W_{111111}^{-i}\, dV. \tag{5.16}$$

Comparing (5.16) and (5.5), we derive the formula for the mass of W-bosons:

$$m_W = \sqrt{\frac{4\, \pi\, (g_2)^2}{m_\chi}\, \frac{e\, v\, \hbar}{c^2}}. \tag{5.17}$$

The term $\mathcal{L}_{44}$ in (5.11) describes the interaction of Z-bosons with the real scalar Higgs field. It is represented by the following two integrals:

$$\mathcal{L}_{44} = \frac{c\, m_Z^2}{4\, \pi\, \hbar^2\, v} \int \sum_{i=0}^{3} \chi\, Z_i\, Z^i\, dV + \frac{c\, m_Z^2}{8\, \pi\, \hbar^2\, v^2} \int \sum_{i=0}^{3} \chi^2\, Z_i\, Z^i\, dV. \tag{5.18}$$

In a similar way, the term $\mathcal{L}_{45}$ in the sum (5.11) describes the interaction of W-bosons with the real scalar Higgs field χ:

$$\begin{aligned} \mathcal{L}_{45} = \frac{c\, m_W^2}{4\, \pi\, \hbar^2\, v} \int \sum_{i=0}^{3} \chi\, W_i^{+111111}\, W_{111111}^{-i}\, dV\, + \\ + \frac{c\, m_W^2}{8\, \pi\, \hbar^2\, v^2} \int \sum_{i=0}^{3} \chi^2\, W_i^{+111111}\, W_{111111}^{-i}\, dV. \end{aligned} \tag{5.19}$$

The term $\mathcal{L}_{52}$ in the sum (5.11) describes the self-action of the Higgs field:

$$\mathcal{L}_{52} = -\frac{m_\chi\,c}{2\,v}\int \chi^3\,dV - \frac{m_\chi\,c}{2\,v^2}\int \chi^4\,dV. \tag{5.20}$$

The last term $\mathcal{L}_{53}$ in the sum (5.11) is a constant term:

$$\mathcal{L}_{53} = \frac{m_\chi\,c\,v^2}{4}\int dV. \tag{5.21}$$

Usually constant terms like (5.21) are omitted. However, we prefer to keep it in (5.11), since in a non-flat space-time manifold M it can contribute to the cosmological constant Λ (see [8]).

Unlike the sum (4.44), the interaction and self-action terms (5.18), (5.19), and (5.20) in the sum (5.11) are purely potential terms. They do not comprise the covariant derivatives of the fields $\mathbf{Z}$, $\mathbf{W}^{+}$, $\mathbf{W}^{-}$, and χ.

6. The lepton masses.

Having derived the formulas (5.15) and (5.17) for the masses of Z and W-bosons, now we return back to the leptons represented in the table (1.1). The kinetic term for leptons in the total action of the Standard Model is written as follows:

$$\begin{aligned}
\mathcal{L}_6 = i\,\hbar\int\sum_{i=e,\mu,\tau}\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\times\\
\times\ \text{Đ}_{\alpha\bar\alpha}\,\overline{\overset{\bullet}{\psi}{}^{\bar a\bar\alpha}_{111}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\ \nabla_{q}\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]\,dV\,+\\
+\ i\,\hbar\int\sum_{i=e,\mu,\tau}\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\times\\
\times\ \text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\overline{\overset{\circ}{\psi}{}^{\bar a}_{111111}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\ \nabla_{q}\overset{\circ}{\psi}{}^{b}_{111111}[i]\,dV.
\end{aligned}\tag{6.1}$$

Mass terms corresponding to (6.1) should yield masses for the electron, muon, and tauon, but their neutrinos should remain massless. For this reason mass terms for leptons are introduced through the interaction with the Higgs field:

$$\begin{aligned}
\mathcal{L}_7 = -\sum_{i=e,\mu,\tau}h[i]\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\times\\
\times\ \text{Đ}_{\alpha\bar\alpha}\,D_{a\bar a}\,\overline{\overset{\circ}{\psi}{}^{\bar a}_{111111}[i]}\ \overline{\varphi^{\bar\alpha 111}}\ \overset{\bullet}{\psi}{}^{a\alpha}_{111}[i]\,dV\,-\\
-\sum_{i=e,\mu,\tau}h[i]\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\times\\
\times\ \text{Đ}_{\alpha\bar\alpha}\,D_{a\bar a}\,\overline{\overset{\bullet}{\psi}{}^{\bar a\bar\alpha}_{111}[i]}\ \varphi^{\alpha 111}\ \overset{\circ}{\psi}{}^{a}_{111111}[i]\,dV.
\end{aligned}\tag{6.2}$$

Note that in (6.1) and (6.2) we used both doublet and singlet wave functions (1.4) and (1.5). By $h[i]$ in (6.2) we denote three real constants $h[e]$, $h[\mu]$, $h[\tau]$ specific for each lepton generation.

Now remember the projection operators $\mathbf{P}$ and $\mathbf{Q}$ introduced by the formulas (4.3) and (4.8). Applying them to doublet parts of lepton wave functions and taking into account the equality (4.12), we get the expansion

$$\overset{\bullet}{\psi}{}^{a\alpha}_{111}[i]=\overset{\bullet}{\psi}{}^{a}_{111111}[i]\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\overset{\bullet}{\psi}{}^{a}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}. \tag{6.3}$$

Note that $\overset{\bullet}{\psi}{}^{a}_{111111}[i]$ in (6.3) are similar to $\overset{\circ}{\psi}{}^{a}_{111111}[i]$ in (6.1) and (6.2). These functions are chiral and antichiral parts of the complete wave functions:

$$\psi^{a}_{111111}[i]=\overset{\bullet}{\psi}{}^{a}_{111111}[i]+\overset{\circ}{\psi}{}^{a}_{111111}[i]. \tag{6.4}$$

The singlet wave functions with the components (6.4) describe charged leptons: an electron for $i=e$, a muon for $i=\mu$, and a tauon for $i=\tau$.

The quantities $\overset{\bullet}{\psi}{}^{a}[i]$ are the components of other three singlet wave functions. They describe neutral leptons: a e-neutrino for $i=e$, a μ-neutrino for $i=\mu$, and a τ-neutrino for $i=\tau$. These wave functions are chiral because antichiral (right) neutrinos are not considered in the Standard Model.

A remark. Charged leptons are distinguished from their neutral counterparts due to the nontrivial Higgs vacuum that breaks the initial $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry.

The covariant derivatives ∇_q in (6.1) are complete non-vacuum covariant derivatives. They are evaluated according to the formulas (2.3) and (2.4), where the electro-weak connection components are taken from (2.36) and (2.37) including the components of the gauge fields $\mathbf{A}$ and $\boldsymbol{\mathcal{A}}$. Passing from non-vacuum to vacuum covariant derivatives in the formulas (2.3) and (2.4), we get

$$\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]\to\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]-\frac{i\,e\,g_2}{\hbar\,c}\sum^{2}_{\theta=1}\mathcal{A}^{\alpha}_{q\theta}\,\overset{\bullet}{\psi}{}^{b\theta}_{111}[i]+\frac{3\,i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^{1}_{q1}\,\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i], \tag{6.5}$$

$$\nabla_q\,\overset{\circ}{\psi}{}^{b}_{111111}[i]\to\nabla_q\,\overset{\circ}{\psi}{}^{b}_{111111}[i]+\frac{6\,i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^{1}_{q1}\,\overset{\circ}{\psi}{}^{b}_{111111}[i]. \tag{6.6}$$

Before substituting (6.5) and (6.6) back into (6.1) let's apply (4.35) and (6.3) to (6.5). As a result we get the following expression:

$$\begin{gathered}\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]\to\nabla_q\overset{\bullet}{\psi}{}^{b}_{111111}[i]\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\nabla_q\overset{\bullet}{\psi}{}^{b}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}-\\ -\frac{i\,e\,g_2}{\hbar\,c}\left(\frac{3\,g_1\,A_q-g_2\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\left(\overset{\bullet}{\psi}{}^{b}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}-\overset{\bullet}{\psi}{}^{b}_{111111}[i]\,\cdot\right.\right.\\ \left.\cdot\,\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}\right)+W^{+111111}_{q}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}+W^{-}_{q111111}\,\times\end{gathered}$$

$$\times\overset{\bullet}{\psi}{}^{b}[i]\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}\biggr)+\frac{3\,i\,e\,g_1}{\hbar\,c}\,\frac{g_2\,A_q+3\,g_1\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\times$$

$$\times\left(\overset{\bullet}{\psi}{}^{b}_{111111}[i]\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\overset{\bullet}{\psi}{}^{b}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}\right).$$

In deriving the above expression for $\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]$ we used the formula (4.33) for $\mathcal{A}^{1}_{k1}$ and the formula (4.35) for $\mathcal{A}^{\alpha}_{q\theta}$. Recollecting terms in it, we get

$$\begin{gathered}
\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]\to\Biggl(\nabla_q\overset{\bullet}{\psi}{}^{b}_{111111}[i]+\frac{i\,e\,g_2}{\hbar\,c}\,\frac{3\,g_1\,A_q-g_2\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]+\\
+\frac{3\,i\,e\,g_1}{\hbar\,c}\,\frac{g_2\,A_q+3\,g_1\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]-\frac{i\,e\,g_2}{\hbar\,c}\,W^{-}_{q111111}\,\overset{\bullet}{\psi}{}^{b}[i]\Biggr)\cdot\\
\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\Biggl(\nabla_q\overset{\bullet}{\psi}{}^{b}[i]-\frac{i\,e\,g_2}{\hbar\,c}\,\frac{3\,g_1\,A_q-g_2\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\bullet}{\psi}{}^{b}[i]+\frac{3\,i\,e\,g_1}{\hbar\,c}\times\\
\times\frac{g_2\,A_q+3\,g_1\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\bullet}{\psi}{}^{b}[i]-\frac{i\,e\,g_2}{\hbar\,c}\,W^{+111111}_{q}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]\Biggr)\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}.
\end{gathered}\tag{6.7}$$

Simplifying the right hand side of the formula (6.7) a little bit more, we find

$$\begin{gathered}
\nabla_q\overset{\bullet}{\psi}{}^{b\alpha}_{111}[i]\to\Biggl(\nabla_q\overset{\bullet}{\psi}{}^{b}_{111111}[i]+\frac{i\,e}{\hbar\,c}\,\frac{6\,g_1\,g_2\,A_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]+\\
+\frac{i\,e}{\hbar\,c}\,\frac{(3\,g_1)^2-(g_2)^2}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,Z_q\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]-\frac{i\,e\,g_2}{\hbar\,c}\,W^{-}_{q111111}\,\overset{\bullet}{\psi}{}^{b}[i]\Biggr)\cdot\\
\cdot\frac{\varphi^{\alpha 111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\biggl(\nabla_q\overset{\bullet}{\psi}{}^{b}[i]+\frac{i\,e}{\hbar\,c}\,\sqrt{(g_2)^2+(3\,g_1)^2}\,Z_q\,\overset{\bullet}{\psi}{}^{b}[i]-\\
-\frac{i\,e\,g_2}{\hbar\,c}\,W^{+111111}_{q}\,\overset{\bullet}{\psi}{}^{b}_{111111}[i]\biggr)\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}.
\end{gathered}\tag{6.8}$$

Acting in a similar way, from (6.6) we derive the following formula:

$$\begin{gathered}
\nabla_q\,\overset{\circ}{\psi}{}^{b}_{111111}[i]\to\nabla_q\,\overset{\circ}{\psi}{}^{b}_{111111}[i]+\frac{i\,e}{\hbar\,c}\,\frac{6\,g_1\,g_2\,A_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\times\\
\times\overset{\circ}{\psi}{}^{b}_{111111}[i]+\frac{i\,e}{\hbar\,c}\,\frac{18\,(g_1)^2}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,Z_q\,\overset{\circ}{\psi}{}^{b}_{111111}[i].
\end{gathered}\tag{6.9}$$

The expansion (6.4) can be teated as the expansion of lepton wave functions into chiral and antichiral parts. Therefore, using (1.10), we write

$$\overset{\bullet}{\psi}{}^{b}_{111111}[i]=\sum_{c=1}^{2}\overset{\bullet}{H}{}^{b}_{c}\;\psi^{c}_{111111}[i],\qquad\qquad \overset{\circ}{\psi}{}^{b}_{111111}[i]=\sum_{c=1}^{2}\overset{\circ}{H}{}^{b}_{c}\;\psi^{c}_{111111}[i].\tag{6.10}$$

Here $\overset{\bullet}{H}{}^{b}_{c}$ and $\overset{\circ}{H}{}^{b}_{c}$ are the components of the chiral and antichiral projection operators introduced in (1.9) and (1.10).

Now let's substitute (6.8) and (6.9) into (6.1). As a result, using the formulas (2.64), (4.6), (4.7) and the orthogonality condition (4.10), we derive

$$\mathcal{L}_6=\mathcal{L}_{61}+\mathcal{L}_{62}+\mathcal{L}_{63}+\mathcal{L}_{64}+\mathcal{L}_{65}.\tag{6.11}$$

The first term $\mathcal{L}_{61}$ in the right hand side of the formula (6.11) is a standard kinetic term for three spin 1/2 particles with electric charge Q in an electromagnetic field:

$$\begin{gathered}\mathcal{L}_{61}=i\,\hbar\int\sum_{i=e,\mu,\tau}\;\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\,\text{Đ}^{11}\times\\ \times\;\overline{\psi^{\bar a}_{111111}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\left(\nabla_{q}\,\psi^{b}_{111111}[i]-\frac{i\,Q}{\hbar\,c}\,A_{q}\,\psi^{b}_{111111}[i]\right)dV.\end{gathered}\tag{6.12}$$

The electric charge Q for all charged leptons is given by the following formula:

$$Q=-\frac{6\,e\,g_1\,g_2}{\sqrt{(g_2)^2+(3\,g_1)^2}}.\tag{6.13}$$

Since one of the three particles described by (6.12) is an electron, its charge $Q=-e$. Therefore, from (6.13) we derive the following equality relating g_1 and g_2:

$$\frac{6\,g_1\,g_2}{\sqrt{(g_2)^2+(3\,g_1)^2}}=1.\tag{6.14}$$

Due to (6.13) and (6.14) the electric charge of all three charged leptons e, μ, and τ in the table (1.1) is negative and is equal to the charge of an electron:

$$Q=-e.\tag{6.15}$$

The term $\mathcal{L}_{62}$ in (6.11) is a standard kinetic term describing three electrically neutral leptons — e-neutrino, μ-neutrino, abd τ-neutrino:

$$\mathcal{L}_{62}=i\,\hbar\int\sum_{i=e,\mu,\tau}\;\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\overline{\overset{\bullet}{\psi}{}^{\bar a}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\,\nabla_{q}\overset{\bullet}{\psi}{}^{b}[i]\,dV.\tag{6.16}$$

Note that the wave functions of neutrinos (6.16) have only chiral components. Their antichiral components are zero. This is the sign of chiral-to-antichiral asymmetry of the Standard Model.

The term $\mathcal{L}_{63}$ in (6.11) is a purely potential term. It describes the interaction of charged leptons with Z-bosons. Applying (6.10), we write

$$\begin{gathered}\mathcal{L}_{63}=-\frac{e}{c}\int\sum_{i=e,\mu,\tau}\sum^{4}_{a=1}\sum^{4}_{\bar{a}=1}\sum^{4}_{b=1}\sum^{4}_{c=1}\sum^{3}_{q=0}D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\times\\ \times\overline{\psi^{\bar{a}}_{111111}[i]}\,D_{a\bar{a}}\,\gamma^{aq}_{c}\,Z_q\,\frac{((3\,g_1)^2-(g_2)^2)\,\overset{\bullet}{H}{}^{c}_{b}+18\,(g_1)^2\,\overset{\circ}{H}{}^{c}_{b}}{\sqrt{(g_2)^2+(3\,g_1)^2}}\ \psi^{b}_{111111}[i]\,dV.\end{gathered}\tag{6.17}$$

The presence of the components of chiral and anichiral projectors (1.9) in (6.17) is another sign of chiral-to-antichiral asymmetry of the Standard Model.

The term $\mathcal{L}_{64}$ in (6.11) is a purely potential term. It describes the interaction of electrically neutral leptons with Z-bosons. From (6.8) we derive

$$\begin{gathered}\mathcal{L}_{64}=-\frac{e}{c}\int\sum_{i=e,\mu,\tau}\sum^{4}_{a=1}\sum^{4}_{\bar{a}=1}\sum^{4}_{b=1}\sum^{4}_{c=1}\sum^{3}_{q=0}\overline{\psi^{\bar{a}}[i]}\,D_{a\bar{a}}\times\\ \times\gamma^{aq}_{c}\,Z_q\sqrt{(g_2)^2+(3\,g_1)^2}\,\overset{\bullet}{H}{}^{c}_{b}\ \psi^{b}[i]\,dV.\end{gathered}\tag{6.18}$$

Like (6.17), this term (6.18) also breaks the chiral-to-antichiral symmetry.

The term $\mathcal{L}_{65}$ in the sum (6.11) is also a purely potential term. It describes the interaction of charged and neutral leptons with W-bosons:

$$\begin{gathered}\mathcal{L}_{65}=\frac{e}{c}\int\sum_{i=e,\mu,\tau}\sum^{4}_{a=1}\sum^{4}_{\bar{a}=1}\sum^{4}_{b=1}\sum^{3}_{q=0}D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\,D^{11}\times\\ \times\overline{\psi^{\bar{a}}_{111111}[i]}\,D_{a\bar{a}}\,\gamma^{aq}_{b}\,g_2\,W^{-}_{q111111}\,\overset{\bullet}{\psi}{}^{b}[i]\ dV+\\ +\frac{e}{c}\int\sum_{i=e,\mu,\tau}\sum^{4}_{a=1}\sum^{4}_{\bar{a}=1}\sum^{4}_{b=1}\sum^{3}_{q=0}\overline{\overset{\bullet}{\psi}{}^{\bar{a}}[i]}\,D_{a\bar{a}}\,\gamma^{aq}_{b}\,g_2\,W^{+111111}_{q}\,\psi^{b}_{111111}[i]\ dV.\end{gathered}\tag{6.19}$$

The integral $\mathcal{L}_{65}$ in (6.19) has the same sign of chiral-to-antichiral asymmetry of the Standard Model as the integral (6.16).

Now let's proceed with the integral (6.2). It is a purely potential action integral. Substituting (6.3) into (6.2), we take into account (2.64), (4.6), (4.7), and the orthogonality condition (4.10). Apart from those mentioned above, we take into account the formula (3.12). As a result we get

$$\mathcal{L}_7=\mathcal{L}_{71}+\mathcal{L}_{72}.\tag{6.20}$$

The first term $\mathcal{L}_{71}$ in the expansion (6.20) is a purely potential term. Moreover, it is a mass term. It determines the masses of three charged leptons — an electron, a muon, and a tauon in the leptons generation table (1.1):

$$\mathcal{L}_{71}=-\sum_{i=e,\mu,\tau}\frac{h[i]\,v}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\times$$
$$\times\,D_{a\bar a}\,\overline{\psi^{\bar a}_{111111}[i]}\,\psi^{a}_{111111}[i]\,dV. \tag{6.21}$$

The second term $\mathcal{L}_{72}$ in (6.20) is very similar to (6.21). It is also a purely potential term describing the interaction of charged leptons with the real scalar Higgs field:

$$\mathcal{L}_{72}=-\sum_{i=e,\mu,\tau}\frac{h[i]}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\,\not{D}^{11}\times$$
$$\times\,D_{a\bar a}\,\chi\,\overline{\psi^{\bar a}_{111111}[i]}\,\psi^{a}_{111111}[i]\,dV. \tag{6.22}$$

From (6.21) we derive the following formulas for the masses of charged leptons:

$$m_e=\frac{h[e]\,v}{\sqrt{2}\,c}\,,\qquad\qquad m_\mu=\frac{h[\mu]\,v}{\sqrt{2}\,c}\,,\qquad\qquad m_\tau=\frac{h[\tau]\,v}{\sqrt{2}\,c}\,. \tag{6.23}$$

The lepton part of the total action integral is exhausted by (6.21) and (6.22). For this reason uncharged leptons are massless particles in the Standard Model.

7. The quark masses.

Quarks are represented in the table (1.2). Like leptons they are subdivided into three pairs (three generations). Like in the case of leptons, the chiral parts of quark wave functions form SU(2)-doublets, while their antichiral parts are singlets. Here is the kinetic term of the quark action integral:

$$\mathcal{L}_8=i\,\hbar\int\sum_{i=1}^{3}\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}_{11}\times$$
$$\times\;\not{D}_{\alpha\bar\alpha}\,\mathbb{D}_{\beta\bar\beta}\,\overline{\dot\psi^{\bar a1\bar\alpha\bar\beta}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\;\nabla_{q}\dot\psi^{b1\alpha\beta}[i]\;dV\,+$$
$$+\;i\,\hbar\int\sum_{i=u,c,t}\;\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}_{11}\,\not{D}_{11}\,\not{D}_{11}\times$$
$$\times\;\not{D}_{11}\,\not{D}_{\alpha\bar\alpha}\,\mathbb{D}_{\beta\bar\beta}\,\overline{\mathring\psi^{\bar a1111\bar\beta}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\;\nabla_{q}\mathring\psi^{b1111\beta}[i]\,dV\,+ \tag{7.1}$$
$$+\;i\,\hbar\int\sum_{i=d,s,b}\;\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{q=0}^{3}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}^{11}\times$$
$$\times\;\not{D}^{11}\,\not{D}_{\alpha\bar\alpha}\,\mathbb{D}_{\beta\bar\beta}\,\overline{\mathring\psi^{\bar a\bar\beta}_{11}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\;\nabla_{q}\mathring\psi^{b\beta}_{11}[i]\,dV.$$

The mass terms for the quark action integral are more complicated as compared to the case of leptons because of the generation mixing:

$$
\begin{aligned}
\mathcal{L}_9 = &-\sum_{i=1}^{3}\sum_{j=1}^{3} h_1[ij]\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} \not{D}_{11}\,\times\\
&\times\,\not{D}_{\alpha\bar\alpha}\,\not{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\overset{\circ}{\psi}{}^{\bar a1111\bar\beta}[i]}\;\overline{\phi^{\bar\alpha}_{111}}\;\overset{\bullet}{\psi}{}^{a1\alpha\beta}[j]\,dV\,-\\
&-\sum_{i=1}^{3}\sum_{j=1}^{3}\overline{h_1[ji]}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} \not{D}_{11}\,\times\\
&\times\,\not{D}_{\alpha\bar\alpha}\,\not{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\overset{\bullet}{\psi}{}^{\bar a1\bar\alpha\bar\beta}[i]}\;\phi^{\alpha}_{111}\;\overset{\circ}{\psi}{}^{a1111\beta}[j]\,dV\,-\\
&-\sum_{i=1}^{3}\sum_{j=1}^{3} h_2[ij]\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} \not{D}_{11}\,\times\\
&\times\,\not{D}_{\alpha\bar\alpha}\,\not{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\overset{\circ}{\psi}{}^{\bar a\bar\beta}_{11}[i]}\;\overline{\varphi^{\bar\alpha111}}\;\overset{\bullet}{\psi}{}^{a1\alpha\beta}[j]\,dV\,-\\
&-\sum_{i=1}^{3}\sum_{j=1}^{3}\overline{h_2[ji]}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\alpha=1}^{2}\sum_{\bar\alpha=1}^{2}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} \not{D}_{11}\,\times\\
&\times\,\not{D}_{\alpha\bar\alpha}\,\not{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\overset{\bullet}{\psi}{}^{\bar a1\bar\alpha\bar\beta}[i]}\;\varphi^{\alpha111}\;\overset{\circ}{\psi}{}^{a\beta}_{11}[j]\,dV.
\end{aligned}
\tag{7.2}
$$

Like in (6.1) and in (6.2) for leptons, in the above two action integrals for quarks both singlet and doublet wave functions (1.13) and (1.14) are used. But instead of three real parameters $h[e]$, $h[\mu]$, $h[\tau]$ here we have two complex 3×3 matrices of such parameters $h_1[ij]$ and $h_2[ij]$. In the case of leptons we used the expansion (6.3). For quarks such an expansion looks like

$$
\overset{\bullet}{\psi}{}^{a1\alpha\beta}[i]=\overset{\bullet}{\psi}{}^{a\beta}_{11}[i]\cdot\frac{\varphi^{\alpha111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\overset{\bullet}{\psi}{}^{a1111\beta}[i]\cdot\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}.
\tag{7.3}
$$

The chiral coefficients $\overset{\bullet}{\psi}{}^{a\beta}_{11}[i]$ and $\overset{\bullet}{\psi}{}^{a1111\beta}[i]$ from (7.3) are complementary to the antichiral wave functions (1.13) and (1.14). Indeed, we can define

$$
\begin{aligned}
&\psi^{a1111\beta}[i]=\overset{\bullet}{\psi}{}^{a1111\beta}[i]+\overset{\circ}{\psi}{}^{a1111\beta}[i],\\
&\psi^{a\beta}_{11}[i]=\overset{\bullet}{\psi}{}^{a\beta}_{11}[i]+\overset{\circ}{\psi}{}^{a\beta}_{11}[i].
\end{aligned}
\tag{7.4}
$$

The formulas (7.4) are analogous to (6.4). The wave functions introduced by means of the formulas (7.4) are interpreted as the complete wave functions of quarks:

$$
\begin{array}{lll}
\psi^{a1111\beta}[u], & \psi^{a1111\beta}[c], & \psi^{a1111\beta}[t], \\[1ex]
\psi^{a\beta}_{11}[d], & \psi^{a\beta}_{11}[s], & \psi^{a\beta}_{11}[b].
\end{array}
\tag{7.5}
$$

The first line in (7.5) corresponds to upper level quarks, i. e. an up-quark, a charm-quark, and a top-quark, the second line describes lower level quarks — a down-quark, a strange-quark, and a bottom-quark.

The covariant derivatives ∇_q in (7.1) are complete non-vacuum covariant derivatives. They are evaluated according to the formulas (2.5), (2.6), and (2.7), where the electro-weak connection components are taken from (2.36), (2.37), and (2.38) including the components of the gauge fields $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{A}}$, and $\boldsymbol{\mathcal{A}}$. Passing from non-vacuum to vacuum covariant derivatives in (2.5), (2.6), and (2.7) we get

$$
\begin{gathered}
\nabla_q\overset{\bullet}{\psi}{}^{a1\alpha\beta}[i] \to \nabla_q\overset{\bullet}{\psi}{}^{a1\alpha\beta}[i] - \frac{i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^1_{q1}\,\overset{\bullet}{\psi}{}^{a1\alpha\beta}[i] - \\
-\ \frac{i\,e\,g_2}{\hbar\,c}\sum^2_{\theta=1}\boldsymbol{\mathcal{A}}^\alpha_{q\theta}\,\overset{\bullet}{\psi}{}^{a1\theta\beta}[i] - \frac{i\,e\,g_3}{\hbar\,c}\sum^3_{\theta=1}\boldsymbol{\mathcal{A}}^\alpha_{q\theta}\,\overset{\bullet}{\psi}{}^{a1\alpha\theta}[i],
\end{gathered}
\tag{7.6}
$$

$$
\begin{gathered}
\nabla_q\overset{\circ}{\psi}{}^{a1111\beta}[i] \to \nabla_q\overset{\circ}{\psi}{}^{a1111\beta}[i] - \\
-\ \frac{4\,i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^1_{q1}\,\overset{\circ}{\psi}{}^{a1111\beta}[i] - \frac{i\,e\,g_3}{\hbar\,c}\sum^3_{\theta=1}\boldsymbol{\mathcal{A}}^\alpha_{q\theta}\,\overset{\circ}{\psi}{}^{a1111\theta}[i],
\end{gathered}
\tag{7.7}
$$

$$
\begin{gathered}
\nabla_q\overset{\circ}{\psi}{}^{a\beta}_{11}[i] \to \nabla_q\overset{\circ}{\psi}{}^{a\beta}_{11}[i] + \frac{2\,i\,e\,g_1}{\hbar\,c}\,\mathcal{A}^1_{q1}\,\overset{\circ}{\psi}{}^{a\beta}_{11}[i] - \\
-\ \frac{i\,e\,g_3}{\hbar\,c}\sum^3_{\theta=1}\mathbb{A}^\alpha_{q\theta}\,\overset{\circ}{\psi}{}^{a\theta}_{11}[i].
\end{gathered}
\tag{7.8}
$$

The next step is to apply the formulas (4.33) and (4.35) to (7.6), (7.7), and (7.8). Let's begin with the last two formulas (7.7) and (7.8) for SU(2)-singlet functions. In the case of the formula (7.7), applying (4.33) to it, we derive

$$
\begin{gathered}
\nabla_q\overset{\circ}{\psi}{}^{a1111\beta}[i] \to \nabla_q\overset{\circ}{\psi}{}^{a1111\beta}[i] - \frac{i\,e}{\hbar\,c}\,\frac{4\,g_1\,g_2\,A_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\circ}{\psi}{}^{a1111\beta}[i] - \\
-\ \frac{i\,e}{\hbar\,c}\,\frac{12\,(g_1)^2\,Z_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\circ}{\psi}{}^{a1111\beta}[i] - \frac{i\,e\,g_3}{\hbar\,c}\sum^3_{\theta=1}\boldsymbol{\mathcal{A}}^\alpha_{q\theta}\,\overset{\circ}{\psi}{}^{a1111\theta}[i].
\end{gathered}
\tag{7.9}
$$

In a similar way, applying (4.33) to the formula (7.8), we derive

$$
\begin{gathered}
\nabla_q\overset{\circ}{\psi}{}^{a\beta}_{11}[i] \to \nabla_q\overset{\circ}{\psi}{}^{a\beta}_{11}[i] + \frac{i\,e}{\hbar\,c}\,\frac{2\,g_1\,g_2\,A_k}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\overset{\circ}{\psi}{}^{a\beta}_{11}[i] + \\
+\ \frac{i\,e}{\hbar\,c}\,\frac{6\,(g_1)^2}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,Z_q\,\overset{\circ}{\psi}{}^{a\beta}_{11}[i] - \frac{i\,e\,g_3}{\hbar\,c}\sum^3_{\theta=1}\mathbb{A}^\alpha_{q\theta}\,\overset{\circ}{\psi}{}^{a\theta}_{11}[i].
\end{gathered}
\tag{7.10}
$$

Applying (4.33) and (4.35) to (7.6), we should take into account the formula (7.3):

$$
\begin{gathered}
\nabla_q\dot{\psi}^{a1\alpha\beta}[i]\to\left(\nabla_q\dot{\psi}^{a\beta}_{11}[i]+\frac{i\,e}{\hbar\,c}\,\frac{2\,g_1\,g_2\,A_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\dot{\psi}^{a\beta}_{11}[i]-\right.\\
-\frac{i\,e}{\hbar\,c}\,\frac{(3\,(g_1)^2+(g_2)^2)}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,Z_q\,\dot{\psi}^{a\beta}_{11}[i]-\,\frac{i\,e\,g_2}{\hbar\,c}\,W^{-}_{q111111}\,\dot{\psi}^{a1111\beta}[i]-\\
\left.-\frac{i\,e\,g_3}{\hbar\,c}\,\sum^{3}_{\theta=1}\mathcal{A}^{\,\alpha}_{q\theta}\,\dot{\psi}^{a\theta}_{11}[i]\right)\cdot\,\frac{\varphi^{\alpha111}[vac]}{|\boldsymbol{\varphi}[vac]|}+\left(\nabla_q\dot{\psi}^{a1111\beta}[i]-\frac{i\,e}{\hbar\,c}\,\times\right.\\
\times\,\frac{4\,g_1\,g_2\,A_q}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,\dot{\psi}^{a1111\beta}[i]-\frac{i\,e}{\hbar\,c}\,\frac{3\,(g_1)^2-(g_2)^2}{\sqrt{(g_2)^2+(3\,g_1)^2}}\,Z_q\,\dot{\psi}^{a1111\beta}[i]-\\
\left.-\frac{i\,e\,g_2}{\hbar\,c}\,W^{+111111}_{q}\,\dot{\psi}^{a\beta}_{11}[i]-\frac{i\,e\,g_3}{\hbar\,c}\,\sum^{3}_{\theta=1}\mathcal{A}^{\,\alpha}_{q\theta}\,\dot{\psi}^{a1111\theta}[i]\right)\cdot\,\frac{\phi^{\alpha}_{111}[vac]}{|\boldsymbol{\phi}[vac]|}.
\end{gathered}
\tag{7.11}
$$

The formula (7.11) is analogous to (6.8). When substituting (7.9), (7.10), and (7.11) into (7.1) we take into account the equality (6.14). Then we get

$$
\mathcal{L}_8=\mathcal{L}_{81}+\mathcal{L}_{82}+\mathcal{L}_{83}+\mathcal{L}_{84}+\mathcal{L}_{85}.
\tag{7.12}
$$

The first term $\mathcal{L}_{81}$ in the right hand side of the formula (7.12) is a standard kinetic term for three spin 1/2 particles in an electromagnetic field:

$$
\begin{gathered}
\mathcal{L}_{81}=i\,\hbar\int\sum_{i=d,s,b}\,\sum^{4}_{a=1}\sum^{4}_{\bar a=1}\sum^{4}_{b=1}\sum^{3}_{q=0}\sum^{3}_{\beta=1}\sum^{3}_{\bar\beta=1}D^{11}\,D^{11}\,\text{\DJ}_{\beta\bar\beta}\,\overline{\psi^{\bar a\bar\beta}_{11}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\,\times\\
\times\left(\nabla_q\psi^{b\beta}_{11}[i]+\frac{i\,e}{3\,\hbar\,c}\,A_q\,\psi^{b\beta}_{11}[i]-\frac{i\,e\,g_3}{\hbar\,c}\,\sum^{3}_{\theta=1}\mathcal{A}^{\,\alpha}_{q\theta}\,\psi^{b\theta}_{11}[i]\right)dV.
\end{gathered}
\tag{7.13}
$$

The second term $\mathcal{L}_{82}$ is a standard kinetic term for other three particles

$$
\begin{gathered}
\mathcal{L}_{82}=i\,\hbar\int\sum_{i=u,c,t}\,\sum^{4}_{a=1}\sum^{4}_{\bar a=1}\sum^{4}_{b=1}\sum^{3}_{q=0}\sum^{3}_{\beta=1}\sum^{3}_{\bar\beta=1}D^{11}\,D^{11}\,\times\\
\times\,D^{11}\,D^{11}\,\text{\DJ}_{\beta\bar\beta}\,\overline{\psi^{\bar a1111\bar\beta}[i]}\,D_{a\bar a}\,\gamma^{aq}_{b}\,\left(\nabla_q\psi^{b1111\beta}[i]\,-\right.\\
\left.-\frac{2\,i\,e}{3\,\hbar\,c}\,A_q\,\psi^{b1111\beta}[i]-\frac{i\,e\,g_3}{\hbar\,c}\,\sum^{3}_{\theta=1}\mathcal{A}^{\,\alpha}_{q\theta}\,\psi^{b1111\theta}[i]\right)dV.
\end{gathered}
\tag{7.14}
$$

Looking at (7.14), we see that all upper level quarks, i. e. an up-quark, a charm-quark, and a top-quark (see table (1.2)) have the same positive electric charge

$$Q = +\frac{2}{3}\,e. \tag{7.15}$$

Similarly, looking at (7.13), we find that all lower level quarks, i. e. a down-quark, a strange-quark, and a bottom-quark have the same negative electric charge

$$Q = -\frac{1}{3}\,e. \tag{7.16}$$

Compare (7.15) and (7.16) with (6.15) in the case of charged leptons e, μ, and τ.

The term $\mathcal{L}_{83}$ in (7.12) is a purely potential term. It describes the interaction of lower level quarks with Z-bosons. From (7.10) and (7.11) we derive

$$\begin{gathered}
\mathcal{L}_{83} = -\frac{e}{c}\int\sum_{i=d,s,b}\,\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{c=1}^{4}\sum_{q=0}^{3}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} D^{11}\,D^{11}\,\mathbf{D}_{\beta\bar\beta}\,\times\\
\times\,\overline{\psi_{11}^{\bar a\bar\beta}[i]}\,D_{a\bar a}\,\gamma_c^{aq}\,Z_q\,\frac{6\,(g_1)^2\,\overset{\circ}{H}{}_b^c - (3\,(g_1)^2 + (g_2)^2)\,\overset{\bullet}{H}{}_b^c}{\sqrt{(g_2)^2 + (3\,g_1)^2}}\;\psi_{11}^{b\beta}[i]\,dV.
\end{gathered}\tag{7.17}$$

The term $\mathcal{L}_{84}$ in (7.12) is also a purely potential term. It describes the interaction of lower upper quarks with Z-bosons. From (7.9) and (7.11) we derive

$$\begin{gathered}
\mathcal{L}_{84} = -\frac{e}{c}\int\sum_{i=d,s,b}\,\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{c=1}^{4}\sum_{q=0}^{3}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} D^{11}\,D^{11}\,\times\\
\times\,D^{11}\,D^{11}\,\mathbf{D}_{\beta\bar\beta}\;\overline{\psi^{\bar a1111\bar\beta}[i]}\;D_{a\bar a}\,\gamma_c^{aq}\,Z_q\,\times\\
\times\,\frac{-12\,(g_1)^2\,\overset{\circ}{H}{}_b^c - (3\,(g_1)^2 - (g_2)^2)\,\overset{\bullet}{H}{}_b^c}{\sqrt{(g_2)^2 + (3\,g_1)^2}}\;\psi^{\,b1111\beta}[i]\,dV.
\end{gathered}\tag{7.18}$$

The term $\mathcal{L}_{85}$ in (7.12) is a purely potential term describing the interaction of upper and lower level quarks with W-bosons. From (7.11) we derive

$$\begin{gathered}
\mathcal{L}_{85} = \frac{e}{c}\int\sum_{i=1}^{3}\,\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{c=1}^{4}\sum_{q=0}^{3}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} D^{11}\,D^{11}\,\mathbf{D}_{\beta\bar\beta}\,\times\\
\times\,\overline{\psi_{11}^{\bar a\bar\beta}[i]}\,D_{a\bar a}\,\gamma_c^{aq}\,g_2\,W_{q111111}^{-}\,\overset{\bullet}{H}{}_b^c\;\psi^{b1111\beta}[i]\,dV\,+\\
+\frac{e}{c}\int\sum_{i=1}^{3}\,\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{c=1}^{4}\sum_{q=0}^{3}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3} D^{11}\,D^{11}\,D^{11}\,\times\\
\times\,D^{11}\,\mathbf{D}_{\beta\bar\beta}\;\overline{\psi^{\bar a1111\bar\beta}[i]}\,D_{a\bar a}\,\gamma_c^{aq}\,g_2\,W_q^{+111111}\,\overset{\bullet}{H}{}_b^c\;\psi_{11}^{b\beta}[i]\,dV.
\end{gathered}\tag{7.19}$$

The quark terms (7.17), (7.18), and (7.19) in the action integral are analogous to the terms (6.17), (6.18), and (6.19) in the case of leptons. Note that the terms responsible for interaction of quarks and gluons are comprised within the kinetic terms (7.13) and (7.14). This means that the color symmetry SU(3) is not broken.

Now let's proceed with the integrals (7.2). They are responsible for the masses of quarks. Substituting (7.3) into (7.2), we derive

$$\mathcal{L}_9=\mathcal{L}_{91}+\mathcal{L}_{92}. \tag{7.20}$$

The first term $\mathcal{L}_{91}$ in the sum (7.20) is written as follows:

$$\begin{aligned}\mathcal{L}_{91}=&-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{v}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}_{11}\,\not{D}_{11}\,\not{D}_{11}\,\not{D}_{11}\,\times\\ &\times\,\not{\mathbb{D}}_{\beta\bar\beta}\,D_{a\bar a}\left(h_1[ij]\,\overset{\bullet}{H}{}^a_b+\overline{h_1[j\,i]}\,\overset{\circ}{H}{}^a_b\right)\overline{\psi^{\bar a1111\bar\beta}[i]}\;\psi^{b1111\beta}[j]\,dV\,-\\ &-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{v}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}^{11}\,\not{D}^{11}\,\times\\ &\times\,\not{\mathbb{D}}_{\beta\bar\beta}\,D_{a\bar a}\left(h_2[ij]\,\overset{\bullet}{H}{}^a_b+\overline{h_2[j\,i]}\,\overset{\circ}{H}{}^a_b\right)\overline{\psi^{\bar a\bar\beta}_{11}[i]}\;\psi^{b\beta}_{11}[j]\,dV.\end{aligned}\tag{7.21}$$

The second term $\mathcal{L}_{92}$ in (7.20) is responcible for the interaction of quarks with the real scalar Higgs field χ. It is very similar to (7.21):

$$\begin{aligned}\mathcal{L}_{92}=&-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{1}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}_{11}\,\not{D}_{11}\,\not{D}_{11}\,\not{D}_{11}\,\times\\ &\times\,\not{\mathbb{D}}_{\beta\bar\beta}\,D_{a\bar a}\,\chi\left(h_1[ij]\,\overset{\bullet}{H}{}^a_b+\overline{h_1[j\,i]}\,\overset{\circ}{H}{}^a_b\right)\overline{\psi^{\bar a1111\bar\beta}[i]}\;\psi^{b1111\beta}[j]\,dV\,-\\ &-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{1}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\not{D}^{11}\,\not{D}^{11}\,\times\\ &\times\,\not{\mathbb{D}}_{\beta\bar\beta}\,D_{a\bar a}\,\chi\left(h_2[ij]\,\overset{\bullet}{H}{}^a_b+\overline{h_2[j\,i]}\,\overset{\circ}{H}{}^a_b\right)\overline{\psi^{\bar a\bar\beta}_{11}[i]}\;\psi^{b\beta}_{11}[j]\,dV.\end{aligned}\tag{7.22}$$

The term (7.21) is a mass term for quarks. However, in general case, using it, one cannot prescribe masses to individual quarks. Let's consider a special case, where the diagonal elements of the coupling constants matrices are real constants:

$$h_1[ii]=\overline{h_1[ii]},\qquad\qquad h_2[ii]=\overline{h_1[ii]}. \tag{7.23}$$

In this special case, where the equalities (7.23) are fulfilled, we can write

$$\mathcal{L}_{91}=\mathcal{L}_{91}[diag]+\mathcal{L}_{91}[not\,diag]. \tag{7.24}$$

The diagonal term $\mathcal{L}_{91}[diag]$ in the expansion (7.24) is written as follows:

$$\begin{gathered}\mathcal{L}_{91}[diag]=-\sum_{i=1}^{3}\frac{h_1[ii]\,v}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\times\\ \times\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\psi^{\bar a1111\bar\beta}[i]}\;\psi^{a1111\beta}[i]\,dV-\sum_{i=1}^{3}\frac{h_1[ii]\,v}{\sqrt{2}}\,\times\\ \times\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}^{11}\,D^{11}\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\psi^{\bar a\bar\beta}_{11}[i]}\;\psi^{b\beta}_{11}[i]\,dV.\end{gathered}\tag{7.25}$$

Looking at (7.25), one can prescribe the following masses to individual quarks:

$$\begin{aligned}&m_u=\frac{h_1[11]\,v}{\sqrt{2}\,c}\,, &&m_c=\frac{h_1[22]\,v}{\sqrt{2}\,c}\,, &&m_t=\frac{h_1[33]\,v}{\sqrt{2}\,c}\,,\\ &m_d=\frac{h_2[11]\,v}{\sqrt{2}\,c}\,, &&m_s=\frac{h_2[22]\,v}{\sqrt{2}\,c}\,, &&m_b=\frac{h_2[33]\,v}{\sqrt{2}\,c}\,.\end{aligned}\tag{7.26}$$

The formulas (7.26) are similar to the formulas (6.23) for charged leptons. Note that we can impose a more restrictive condition for the coupling constants than that of (7.23), e. g. we can require them to form Hermitian matrices:

$$h_1[ij]=\overline{h_1[ji]}, \qquad\qquad h_2[ij]=\overline{h_1[ji]}.\tag{7.27}$$

In this case the action integrals (7.21) and (7.22) simplify to

$$\begin{gathered}\mathcal{L}_{91}=-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{h_1[ij]\,v}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\times\\ \times\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\psi^{\bar a1111\bar\beta}[i]}\;\psi^{b1111\beta}[j]\,dV-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{h_1[ij]\,v}{\sqrt{2}}\,\times\\ \times\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}^{11}\,D^{11}\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\overline{\psi^{\bar a\bar\beta}_{11}[i]}\;\psi^{b\beta}_{11}[j]\,dV,\end{gathered}\tag{7.28}$$

$$\begin{gathered}\mathcal{L}_{92}=-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{h_1[ij]}{\sqrt{2}}\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\mathit{D}_{11}\,\times\\ \times\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\chi\,\overline{\psi^{\bar a1111\bar\beta}[i]}\;\psi^{b1111\beta}[j]\,dV-\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{h_1[ij]}{\sqrt{2}}\,\times\\ \times\int\sum_{a=1}^{4}\sum_{\bar a=1}^{4}\sum_{b=1}^{4}\sum_{\beta=1}^{3}\sum_{\bar\beta=1}^{3}\mathit{D}^{11}\,D^{11}\,\mathit{D}_{\beta\bar\beta}\,D_{a\bar a}\,\chi\,\overline{\psi^{\bar a\bar\beta}_{11}[i]}\;\psi^{b\beta}_{11}[j]\,dV.\end{gathered}\tag{7.29}$$

Since (7.27) implies (7.23), the formulas (7.26) for quark masses are valid in this case too. Unlike (7.21) and (7.22), the action integrals (7.28) and (7.29) preserve the chiral-to-antichiral symmetry. However, this makes no difference for the Standard Model in whole since there are many other terms in the total action integral that break this symmetry.

8. Conclusion.

The main purpose of this paper is to explain the Standard Model of elementary particles in a little bit non-standard way different from that traditionally used in physical literature. In two previous papers [2] and [6] three special complex vector bundles over the space-time manifold M were introduced and studied. These bundles provide a geometric background for describing the Standard Model in the case of a non-flat space-time manifold M, i. e. in the presence of a gravitation field. The actual description of the Standard Model in terms of these three bundles is given in the present paper.

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