RELATIVISTIC HARDENING AND SOFTENING OF FAST MOVING SPRINGS.

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Abstract. Two identical relativistic spring-mass systems are considered, one in a frame at rest and the other in a fast moving starship. Their small oscillations are studied in the frame at rest. Comparing the frequencies of these oscillations the relativistic spring hardening and softening laws are derived.

1. Introduction.

Relativistic spring-mass systems are studied in several papers (see [1–5]) with the stress on their anharmonicity and delayed force effects. Our stress is toward transferring spring-mass systems to the background of the new theory of gravity whose name is 3D-brane universe model. This new theory has two versions. The first version is developed using the so-called equidistance postulate (see e-prints [6–11] and conference abstracts [12–16]). In the second version of the theory the equidistance postulate is omitted (see e-prints [17–21] and conference abstracts [22–25]). Therefore the second version is more general and we refer the reader to it rather than to the first version.

Our interest in spring-mass systems is fueled by the realization that they are the simplest tick sources for time measuring devices. Studying them, we can come closer to understanding the nature of time itself.

2. Spring-mass systems oscillating along the motion of a vehicle.

Let's consider inertial coordinates x, y, z, t and two identical spring-mass systems associated with them as shown in Fig. 2.1. The first spring-mass system is

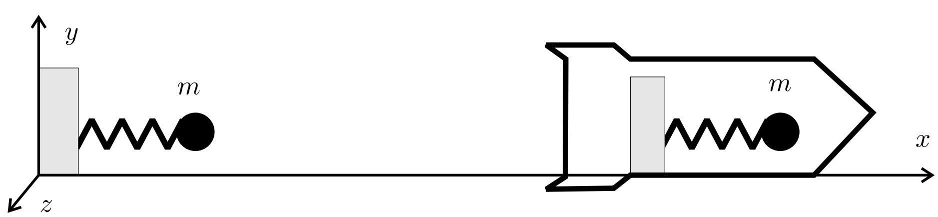

Fig. 2.1

attached to a heavy wall being at rest. The second one is attached to a similar

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heavy wall within a starship moving to the right. Small oscillations of the first spring-mass system are given by the following formula:

$$x(t)=x_0+a\,\sin(\omega\,t)+\ldots\,. \tag{2.1}$$

Small oscillations of the second spring-mass system are given by a similar formula:

$$\tilde{x}(t)=\tilde{x}_0+v\,t+\tilde{a}\,\sin(\tilde{\omega}\,t)+\ldots\,. \tag{2.2}$$

Here v is the speed of the starship. According to [1–5], the oscillations of a relativistic spring-mass system are anharmonic. Therefore we added dots at the end of the formulas (2.1) and (2.2). They present higher order harmonics which are not written explicitly.

The formulas (2.1) and (2.2) determine the coordinates of two point masses m held at the free ends of two springs in Fig. 2.1. Differentiating these formulas with respect to the time variable t, we get

$$v(t)=u\,\cos(\omega\,t)+\ldots\,, \qquad\qquad \tilde{v}(t)=v+\tilde{u}\,\cos(\tilde{\omega}\,t)+\ldots\,, \tag{2.3}$$

where

$$u=a\,\omega, \qquad\qquad \tilde{u}=\tilde{a}\,\tilde{\omega}. \tag{2.4}$$

The quantities $v(t)$ and $\tilde{v}(t)$ in (2.3) are the velocities of the point masses attached to the springs. Their momenta are given by the formulas

$$p(t)=\frac{m\,v(t)}{\sqrt{1-\dfrac{v(t)^2}{c_{\text{br}}^2}}}\,, \qquad\qquad \tilde{p}(t)=\frac{m\,\tilde{v}(t)}{\sqrt{1-\dfrac{\tilde{v}(t)^2}{c_{\text{br}}^2}}} \tag{2.5}$$

(see § 9 of Chapter I in [26]). We used the notation c_{br} for the speed of light in the formulas (2.5) since here it is rather the critical speed for baryonic matter than the speed of electromagnetic waves.

Below we assume the amplitudes of oscillation u and $\tilde{u}$ in the formulas (2.3) to be much smaller than the speed of light:

$$u\ll c_{\text{br}}, \qquad\qquad \tilde{u}\ll c_{\text{br}}. \tag{2.6}$$

However the speed of starship v is not small as compared to the speed of light c_{br}.

Now we proceed to the dynamical equations

$$\frac{dp(t)}{dt}=F(t), \qquad\qquad \frac{d\tilde{p}(t)}{dt}=\tilde{F}(t). \tag{2.7}$$

Substituting (2.3) into (2.5) and taking into account the inequalities (2.6), we derive

$$\frac{dp(t)}{dt}=-m\,u\,\omega\,\sin(\omega\,t)+\ldots\,, \qquad\qquad \frac{d\tilde{p}(t)}{dt}=\frac{-m\,\tilde{u}\,\tilde{\omega}\,\sin(\tilde{\omega}\,t)}{\left(\sqrt{1-\dfrac{v^2}{c_{\text{br}}^2}}\right)^3}+\ldots\,. \tag{2.8}$$

Due to (2.6) in the right hand sides of the dynamical equations (2.7) we should write the elastic restoring forces of the springs in the non-relativistic limit:

$$F(t)=-k\,(x(t)-x_0)+\ldots\,,\qquad\qquad \tilde F(t)=-\tilde k\,(\tilde x(t)-\tilde x_0-v\,t)+\ldots\,. \tag{2.9}$$

Comparing the first equalities in (2.8) and in (2.9), then taking into account (2.1) and (2.4), from (2.7) we derive the classical formula for the frequency ω:

$$\omega=\sqrt{\frac{k}{m}}. \tag{2.10}$$

Similarly, comparing the second equalities in (2.8) and in (2.9), then taking into account (2.1) and (2.4), from (2.7) we derive an equation for the frequency $\tilde\omega$:

$$\frac{\tilde\omega^2}{\left(\sqrt{1-\dfrac{v^2}{c^2_{\text{br}}}}\,\right)^3}=\frac{\tilde k}{m}. \tag{2.11}$$

From the standard special relativity we know that time moves slower onboard a moving starship. From (2.1) we know that $\tilde\omega$ is the tick frequency of the onboard spring-mass system as it is seen for an observer at rest. Therefore $\tilde\omega<\omega$ and these two frequencies are related to each other by the standard relativistic factor:

$$\tilde\omega=\omega\sqrt{1-\frac{v^2}{c^2_{\text{br}}}}. \tag{2.12}$$

Substituting (2.12) into the formula (2.11) and taking into account (2.10), we derive

$$\tilde k=\frac{k}{\sqrt{1-\dfrac{v^2}{c^2_{\text{br}}}}}\,. \tag{2.13}$$

The formula (2.13) expresses the following relativistic spring hardening law.

Theorem 2.1. *According to the formula* (2.13)*, any spring made of a baryonic matter onboard a moving vehicle looks like a more stiff spring for an observer at rest if it is oriented along the direction of the vehicle motion.*

3. Spring-mass systems oscillating perpendicular to the motion of a vehicle.

Again let's consider two identical spring-mass systems. They are shown in Fig. 3.1. The first spring-mass system is attached to a heavy wall being at rest.

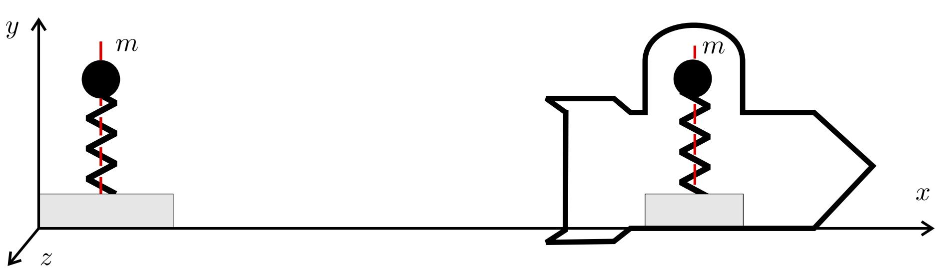

Fig. 3.1

The second one is attached to a similar heavy wall within a starship moving to the right. In this case our spring-mass systems oscillate in the vertical direction perpendicular to the motion of the starship. In order to prevent horizontal deflection of the masses, they are equipped with vertical hard rods. They are shown in red in Fig. 3.1. The masses slide along the rods without friction. Under these conditions their motion is described by the following formulas:

$$x(t)=x_0, \qquad \tilde{x}(t)=\tilde{x}_0+v\,t, \tag{3.1}$$

$$y(t)=y_0+a\,\sin(\omega\,t)+\ldots, \qquad \tilde{y}(t)=\tilde{y}_0+\tilde{a}\,\sin(\tilde{\omega}\,t)+\ldots. \tag{3.2}$$

Differentiating (3.1) and (3.2) with respect to the time variable t, we get

$$v_x(t)=0, \qquad \tilde{v}_x(t)=v, \tag{3.3}$$

$$v_y(t)=u\,\cos(\omega\,t)+\ldots, \qquad \tilde{v}_y(t)=\tilde{u}\,\cos(\tilde{\omega}\,t)+\ldots, \tag{3.4}$$

where

$$u=a\,\omega, \qquad \tilde{u}=\tilde{a}\,\tilde{\omega}. \tag{3.5}$$

The horizontal motion of the masses in this case is constrained by the vertical rods. Therefore the horizontal components of their momenta are of no interest for us. As for the vertical components, they are given by the following formulas:

$$p_y(t)=\frac{m\,v_y(t)}{\sqrt{1-\dfrac{v_y(t)^2}{c_{\text{br}}^2}}}\,, \qquad \tilde{p}_y(t)=\frac{m\,\tilde{v}_y(t)}{\sqrt{1-\dfrac{\tilde{v}_x(t)^2+\tilde{v}_y(t)^2}{c_{\text{br}}^2}}}\,. \tag{3.6}$$

Here again we assume the amplitudes of oscillation u and $\tilde{u}$ in the formulas (3.5) to be much smaller than the speed of light:

$$u\ll c_{\text{br}}, \qquad \tilde{u}\ll c_{\text{br}}. \tag{3.7}$$

But the speed of starship v in (3.3) is not small as compared to the speed of light.

Here are the dynamical equations for the vertical motion of masses:

$$\frac{dp_y(t)}{dt}=F_y(t), \qquad \frac{d\tilde{p}_y(t)}{dt}=\tilde{F}_y(t). \tag{3.8}$$

Substituting (3.3) and (3.4) into (3.6) and taking into account (3.7), we derive

$$\frac{dp_y(t)}{dt}=-m\,u\,\omega\,\sin(\omega\,t)+\ldots, \qquad \frac{d\tilde{p}_y(t)}{dt}=\frac{-m\,\tilde{u}\,\tilde{\omega}\,\sin(\tilde{\omega}\,t)}{\sqrt{1-\dfrac{v^2}{c_{\text{br}}^2}}}+\ldots. \tag{3.9}$$

Due to (3.7) in the right hand sides of the dynamical equations (3.8) we should write the elastic restoring forces of the springs in the non-relativistic limit:

$$F_y(t)=-k\,(y(t)-y_0)+\ldots, \qquad \tilde{F}(t)=-\tilde{k}\,(\tilde{y}(t)-\tilde{y}_0)+\ldots. \tag{3.10}$$

Comparing the first equalities in (3.9) and in (3.10), then taking into account (3.2) and (3.5), from (3.8) we again derive the classical formula for the frequency ω:

$$\omega = \sqrt{\frac{k}{m}}. \tag{3.11}$$

Similarly, comparing the second equalities in (3.9) and in (3.10), then taking into account (3.2) and (3.5), from (3.8) we derive an equation for the frequency $\tilde{\omega}$:

$$\frac{\tilde{\omega}^2}{\sqrt{1-\dfrac{v^2}{c_{\mathrm{br}}^2}}} = \frac{\tilde{k}}{m}. \tag{3.12}$$

The rest is to substitute (2.12) into the formula (3.12) and to take into account (3.11). As a result we derive the relationship

$$\tilde{k} = k\sqrt{1-\frac{v^2}{c_{\mathrm{br}}^2}}. \tag{3.13}$$

The relationship (3.13) expresses the following relativistic spring softening law.

Theorem 3.1. *According to the formula* (3.13)*, any spring made of a baryonic matter onboard a moving vehicle looks like a less stiff spring for an observer at rest if it is oriented perpendicular to the direction of the vehicle motion.*

4. Transferring the results obtained to the new theory of gravity.

Theorems 2.1 and 3.1 are obtained within the standard special relativity. The new theory of gravity initiated in [6] and further developed in [7–11] and [17–21] is different in some aspects. In particular, as an option it admits the existence of a non-baryonic dark matter whose critical speed is different from the speed of light. Therefore, if this dark matter has at least a part that can form stable structures like springs and starships, then the constant c_{br} in Theorems 2.1 and 3.1 for these structures will be replaced by c_{nb}. Through c_{nb} in the new theory the critical speed of the non-baryonic matter is denoted. Actually the non-baryonic matter can be composed by several sorts each with its own critical speed. This option is also compatible with the new theory.

Note that dark matter particles with the critical speed greater than the speed of light were first conjectured by Luis Gonzalez-Mestres in [27]. He gave them the name «superbradyons» in [28].

5. Some comparisons and concluding remarks.

The geometric configuration of springs and masses considered in [1] is similar to that of Section 3 in the present paper. On page 579 of [1] we find the following words: «for the observer at rest the elastic coupling k between the two particles oscillating within the running molecule is weaker than the same coupling in this molecule at rest». These words are in agreement with Theorem 3.1. The results of [2–5] are not comparable to Theorems 2.1 and 3.1 since the problems studied therein are somewhat different from ours.

6. Dedicatory.

This paper is dedicated to my sister Svetlana Abdulovna Sharipova.

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